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Edited for the Council by J. LAURENCE PRITCHARD, Fellow

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NOTICES

Elections

Associate Fellows.—Mr. Ivor Bowen, Mr. R. J. Moffett, Mr. W. D. Murray.

Associates.—Mr. F. Austin, Mr. O. G. Button, Mr. W. J. Dare, Warrant Officer C. W. Goodchild, R.A.F., Mr. A. P. McGowran, P./O. G. P. Mee, Mr. T. Onions, Lieut. K. W. Round, R.A.F., Corporal S. A. Tennant, R.A.F., Sergeant B. A. Vautier, R.A.F.

Students.—Mr. A. B. Carr, Mr. S. A. Seager, Mr. F. W. Westley, P./O. G. L. Worthington, R.A.F.

Members.—Mr. L. Joscelynne-Burnnand, Mr. C. L. G. Colebrook, Capt. W. Mizen.

Donations

The Council desire to acknowledge gratefully the gift of some photographs taken by the late Rev. J. M. Bacon from balloons.

Lecture

A joint meeting of the Society with the Institution of Automobile Engineers will be held in the Theatre of the Royal Society of Arts, 18, John Street, Adelphi, W.C.2, at 7.0 p.m., on Thursday, November 4th, when Mr. G. F. Mucklow will read a paper on "Hydrogen as an Auxiliary Fuel for a Solid Injection Engine." The Institution is kindly providing light refreshments from 6.30 p.m. Colonel the Master of Sempill, Chairman of the Society, will take the chair.

Coventry Branch

The lecture on September 28th, 1926, by Captain F. L. Barnard on "Commercial Aviation," was a great success. An audience of between four and five hundred attended. Two films were shown, the first being the Imperial Airways film illustrating a trip to Paris, and the second showing the Argosy taken from the air. Major F. M. Green, O.B.E., Fellow, presided.

On October 21st Professor B. Melvil Jones read his paper, "Flight of the Future." In spite of the bad weather 150 people were present. Professor Jones, in the course of an exceedingly interesting lecture, pointed out that the Imperial Air Routes were the most important lines of communication to be developed. In his view there was no reasonable future for these islands unless they were knit closely together with the Colonies and the Colonies with each other. He was quite convinced that that was impossible with the present system of transport. The Roman Empire broke down on the question of transport. Coventry had made the latest contribution to the Imperial Air Service by manufacturing the "Argosy." He was enormously impressed with that machine. He hoped to see in his lifetime the institution of a three-day service to Australia, a two-day and a night service to South Africa, a day and two-night service to

India and a day and a bit of a night service to Egypt. That meant travelling at a rate of 150 miles an hour by night and day, practically continuously. A commercial speed of that kind, he believed, was well within the bounds of possibility.

Associate Fellowship Examination

The fourth examination for Associate Fellowship of the Society was held on September 21st, 1926. The papers set are published on page 686 of this number. One candidate only, J. C. Dodds, was successful in both the papers which he submitted. The next examination will take place in the third week of September, 1927.

Paris Aero Show

The Secretary would be glad to hear from any members of the Society who would care to join a party which it is hoped to arrange for the Paris Aero Show in December.

Forthcoming Arrangements

Tuesday, November 2nd, 5.30 p.m.—In the Library, Council Meeting.

Thursday, November 4th, **7.0** p.m.—At the Royal Society of Arts, Joint Meeting with the Institution of Automobile Engineers. Mr. G. F. Mucklow, "Hydrogen as an Auxiliary Fuel for a Solid Injection Engine."

Thursday, November 11th.—In the Library, at 7.0 p.m., Students' Section Inaugural Meeting. Mr. Handley Page, "The Future of Aviation." Lieut.-Colonel I. A. E. Edwards in the chair.

Thursday, November 18th, 6.30 p.m.—At the Royal Society of Arts. Mr. R. S. Capon, "Methods of Performance Testing and Analysis."

Thursday, December 2nd, 6.30 p.m.—At the Royal Society of Arts. Mr. P. B. Henshaw, "Alloy Steels for Aero Work."

J. LAURENCE PRITCHARD, *Hon. Secretary.*

ON THE CALCULATION OF STRESSES IN THE HULLS OF RIGID AIRSHIPS

BY R. V. SOUTHWELL, F.R.S., A.F.R.AE.S.

(*R.38 Memorial Prize Essay*, 1926.)

1 *Introduction. Progress Made in the Period* 1921-1924

An essay which is to be submitted in competition for the R.38 Memorial Prize may fairly presume a knowledge of airship history in recent years, and I shall here touch only on those circumstances which have influenced the writing of this paper. Between the cessation of airship construction in the autumn of 1921 and its revival in the summer of 1924, all branches of official research were discontinued by order of the Air Council, excepting an inquiry into the theoretical aspects of stress-calculation which had been urged by the Aeronautical Research Committee in its report on the disaster to R.38. In December, 1921, an "Airship Stressing Panel" was appointed, with the following terms of reference:—

> "To investigate certain questions relating to the structural strength of airships, and in particular
>
> (1) To consider the validity of the methods which are at present in use, or may have been suggested, for calculating the strength of rigid airships, and
>
> (2) To endeavour to indicate other methods which would enable a closer basis of design to be obtained."

The report of the Airship Stressing Panel was presented in August, 1922. Owing to the cessation of airship construction, no practical application of it was made in this country until 1924; but a paper by Col. V. C. Richmond, read to the International Air Congress of 1923, worked out very fully the consequences of its recommendations, so far as these could be foreseen in the absence of any contact with actual design. In the summer of 1924, when the new designs had just been started and it was proposed to recondition R.33 for experimental work, an "Airworthiness of Airships Panel" was appointed to consider what stipulations should be made in issuing certificates of airworthiness for airships. Its report contained a full discussion of "load factors," which are of course intimately bound up with stress-determination in the general problem of design; but the Panel was not in a position to elaborate the methods of calculation which had been suggested by the Airship Stressing Panel, because the new designs had not progressed far enough to reveal what are the dominating problems of hull construction.

1.1 *Progress Made Since 1924*

I was myself a member of both Panels, and since 1924 it has been my privilege to have close contact with Col. Richmond and his staff at the Royal Airship Works, Cardington, who are engaged on the design of the new "five million airship" R.101: my position has been that of consultant on problems of method encountered in the progress of stress-calculation. The greatly increased size of this new ship, and the ingenuity of the design staff at Cardington, have resulted in many innovations which could not possibly have been foreseen by the Airship Stressing Panel. These innovations (for which no credit can be

claimed by me) will doubtless be described by Col. Richmond at some later date: I am here concerned with them only to the extent that they have modified the standpoint in regard to stress-calculation which, with other members of the Airship Stressing Panel, I held in 1922.

That Panel worked at high pressure, since it had to report within eight months of its appointment; and at a time when it was not possible to maintain contact with actual design, since airship construction in this country, as stated above, had been stopped some months before. As a consequence, certain aspects of stress-calculation were discussed in its report not so much in detail as by implication, and later experience has indicated that, quite apart from the innovations just mentioned, the report might with advantage be supplemented by fuller explanation. It does not, in a word, form a satisfactory manual of airship stressing, and this was inevitable in the circumstances of its compilation.

1.2 *Aim of this Paper*

This paper makes no pretence to supply the need for such a manual: the attempt would in my view be premature while the new designs are still being worked out. But I have thought that it should be possible, and might be useful, to compile an essay of a general and non-detailed nature, forming at once an introduction to, and a continuation of, the report of the Airship Stressing Panel (R. and M. 800). Such an essay might form the framework of a manual to be written in the future, for which such diagrams and formulæ as are relevant would be taken from R. and M. 800: here I have preferred to keep my text relatively clear of formulæ and symbols, by assuming that the reader has that report at hand for reference.

Emphasis should be laid upon the limited aims of the present essay. Stress-calculation is only one part of the general problem of "Design," which is, to decide what type of structure is best fitted to withstand a specified system of loads; and its influence on that decision is of necessity exerted indirectly, since it is impossible to calculate stresses until a type of structure has been assumed. In this paper no attempt is made to deal with the wider question; from the standpoint adopted, both the load distribution and the form of the structure are regarded as specified and not open to question. Thus the paper will not be concerned with Aerodynamics, except in so far as some knowledge of the general nature of the aerodynamic loading must be presumed in a discussion of the conditions for which an airship hull ought to be stressed; nor with those novel features which have recently been evolved, except in so far as these must be taken into account, in order that its discussion of the purely technical problems of stressing may relate to a hull structure of representative design. Such restriction of scope appears to be at once necessary and appropriate: necessary, because the time is not yet ripe for a discussion of problems which are at this moment the subject of deliberation and research; appropriate, because a paper which is not concerned with actual details of airship construction, except by way of example, should thereby acquire a wider application and a more permanent value.

1.3 *Scope of the Paper. Acknowledgments*

The paper consists of three parts. In the first are discussed the general principles and methods of the Theory of Structures, upon which any special method must be based. In the second, the main problem of stress-calculation in airships—viz., the determination of stresses in the hull structure due to the loads imposed by gravity and by aerodynamic forces—is discussed in relation to a representative design; the work of the Airship Stressing Panel is critically reviewed, and the latest available ideas are incorporated in paragraphs intended to form an extension of that Panel's report; some indication is also given of a

standard procedure suitable for routine use in the design office. The third part consists of appendices relating to special problems which have been encountered during the progress of the new designs.

It will perhaps be remarked that few references are made to recent experimental work: this plan has been adopted deliberately,—partly in order to keep the paper within a reasonable length, partly to avoid trespassing on ground which others may desire to traverse in future papers, also intended for the R.38 Memorial Prize Competition. In conclusion, some statement should presumably be made in regard to my claims to originality. I have tried to indicate in the text, or by footnotes, what parts of the essay are based on my own original work; but it will be evident that my ideas have been largely influenced, in a way which makes detailed acknowledgment difficult, by other members of the Panels mentioned in § 1, and more especially by Col. Richmond and the Design Staff at Cardington. Of such influence I desire to express the fullest possible recognition, and it has seemed to me best to state my claims to originality in the form of a list of my own papers—some published, others presented in typed form to the Aeronautical Research Committee—from which I have drawn in the writing of this essay. This list is given in § 5.

2 *General Principles and Methods of the Theory of Structures. The Fundamental Assumption*

The whole science of stress-calculation, as applied to engineering structures, is founded upon the assumption that stress is proportional to strain. This relation ("Hooke's law") is very approximately satisfied by all the usual materials of construction, *within their permissible ranges of stress;* and since the strains which can occur within these permissible ranges are very small, it is possible in the case of actual materials to deduce from it a result which in general would not be a necessary consequence,—*i.e.*, the "Principle of Superposition."

2.1 *The Principle of Superposition*

Upon this principle are based, without exception, all the methods which are employed in stressing elastic structures. It asserts that *the effects of any two systems of load, acting in combination upon a given structure, may be found by calculating the effects of each when acting separately, and superposing the two effects.**

An alternative (and, of course, fundamentally equivalent) statement of the principle is the assertion that a definite "strain-energy function" exists, for any material which obeys Hooke's Law, which is a homogeneous quadratic function of the components of strain. From this statement it follows that the total energy stored as elastic strain in a structure will be expressed by a quadratic function of the displacements of its joints; and if to this strain-energy we add the potentials of all the applied forces, we have the total potential energy of the system under discussion. By a general theorem of Dynamics, this total potential energy must have a minimum value in any configuration of stable equilibrium.

2.2 *"Minimum Energy" Methods for Determining Stresses.* (i) *The Method of Assumed Displacements*

The considerations just stated lead directly to one of the general methods which are available for calculating stresses in engineering structures. Since it has not as yet found much application in practice, it will be sufficient here to

* By "effect" may be understood either stress, strain or displacement.

give one simple illustration of the method, taking as an example a problem of which the solution can be found quite simply in other ways.

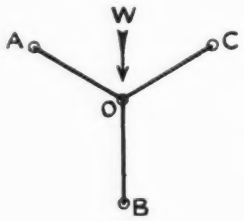

FIG. 1.

OA, OB, OC are three equal rods, hinged at O, A, B, C, and equally inclined to one another. A, B and C are fixed. If a tension T applied to any one rod extends it by one inch, what will be the displacement of O under a load W?

The method now under discussion proceeds by expressing the total potential energy in terms of an arbitrary displacement of the most general type which is possible to the system. The actual displacement is then obtained from the condition that this total energy shall have a minimum value. In the problem just stated, considerations of symmetry show that the displacement will be along OB; if its magnitude is δ inches, the extension in each of OA and OC is $\frac{1}{2}\delta$ inches, and the contraction in OB is δ inches: hence, the total potential energy is given by

$$V = \tfrac{1}{2}T\{(\delta/2)^2 + (\delta/2)^2 + \delta^2\} + \text{const.} - W\delta,$$
$$= \tfrac{3}{4}T\delta^2 + \text{const.} - W\delta,$$

and the condition for a minimum value of V is

$$\partial V/\partial\delta = 3T\delta/2 - W = 0.$$

From this equation we deduce that

$$\delta = 2W/3T,$$

and this result is easily verified.

2.3 *"Minimum Energy Methods" for Determining Stresses.* (ii) *Castigliano's "Method of Least Work"*

An alternative method, which has been widely applied to the calculation of stresses in engineering structures, was propounded by Castigliano in 1873. In this method, we assume *any arbitrary distribution of stresses which satisfies the conditions of equilibrium* (for example, in dealing with complex stress systems in continuous elastic solids, we assume for the stress-components any continuous functions of position which satisfy both the stress-equations of equilibrium and also the imposed boundary conditions), and we form the corresponding expression for the *total energy stored as elastic strain*: the correct solution will be given by the conditions that this total strain-energy (*as contrasted with the total potential energy of* §2.2) shall have a minimum value.

The distinction between this method and that treated in §2.2 will be made clear if we illustrate its application by the same problem as before. We now assume that P is the compression in OB: then for equilibrium $(W - P)$ must be the tension in each of OA and OC, and P/T the contraction of OB,—*i.e.*, the displacement of W, or δ. The total energy stored as elastic strain is therefore

$$U = \tfrac{1}{2} P^2/T + \tfrac{1}{2} (W-P)^2/T + \tfrac{1}{2} (W-P)^2/T,$$
$$= [3P^2/2 - 2WP + W^2] /T,$$

and the condition for a minimum value of U is

$$\partial U/\partial P = (3P - 2W) /T = 0, \quad . \quad . \quad . \quad . \quad (1)$$

whence

$$P = 2W/3,$$

and

$$\delta = P/T = 2W/3T, \text{ as before.}$$

2.31 *The Physical Basis of Castigliano's Theorem*

The "First Theorem of Castigliano," upon which the foregoing method is based, was proved by him, and has generally been proved in text-books on the Theory of Structures, in a way which hardly serves to reveal its physical significance. The fundamental equation of the theorem—viz.,

$$\partial U/\partial P = 0, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

—is obtained merely as a mathematical expression of the equality of two displacements,* and its interpretation as the condition for a minimum value of the strain-energy has the appearance of a somewhat artificial device.

An alternative proof, based on purely physical reasoning, was propounded by me in 1922.† The argument proceeds as follows:—By the Principle of Superposition, the displacements which occur in an elastic structure under any given system of applied loads will have definite values, independent of any stresses which may have existed in the structure initially. Hence, the total work done on the system during the process of loading, as the external forces increase from zero to their specified values, will be unaffected by the initial stresses, and will depend solely upon the form and elastic properties of the structure. If there is initially no external load acting on the structure, so that the initial stresses are a self-equilibrating system arising simply from internal strains, there will be no other forces doing work when the specified loads are applied. Hence, by the principle of Conservation of Energy, the additional strain-energy stored in the structure will be equal to the work done by the specified load-system, and therefore, as we have just seen, it will be definite in amount.

Expressing this result in symbols, we have the equation

$$U = U_0 + W,$$

where U is the total energy stored as elastic strain in the structure, *after* the specified load-system is applied,

U_0 is the total energy stored *initially* as elastic strain in the structure,

and W is the work done in the process of loading (*i.e.*, as the external loads increase from zero to their specified values).

Now in assuming a stress-system which satisfies all the conditions of equilibrium but is otherwise unrestricted (as described in §2.3), we are tacitly assuming the existence of some initial system of internal stress; and in deducing from this the corresponding expression for the total energy stored as elastic strain in the final configuration, we are calculating the quantity U of the foregoing equation. We have seen that this quantity will consist of an invariant part W and of a variable part U_0, which is the strain-energy corresponding to our assumed initial stress-system; and since both U_0 and W, in the nature of the

* The "Method of Least Work" is thus shown to be, in essentials, identical with the "Method of Comparison of Deflections" which is frequently used as an alternative. *Cf.* W. S. Farren, Aeronautical Research Committee R. & M. No. 769.

† *Phil. Mag.*, January, 1923.

case, are necessarily positive,* it follows that U will have its minimum value when U_0 is zero,—*i.e.*, when the structure is initially free from internal strain. Conversely, *the correct solution for a structure which is initially unstrained may be derived from the conditions that U shall have its minimum value.*†

2.4 *" Frames "*

The calculation of stresses in an elastic structure is in general an extremely difficult problem, falling within the province of the Mathematical Theory of Elasticity. In engineering construction, where it is of the utmost importance that our calculations shall be as simple and as reliable as possible, the problem is usually simplified by designing structures in the form of " frames ". A " frame ", in theory, is a structure consisting of various " members " designed to sustain tension or compression, and connected at their extremities by joints which are perfectly free, so that they cannot incur any bending action.

2.41 *The " Skeleton Diagram "*

In most practical cases it is an easy matter to arrange for transference of the more important loads to a fairly small number of isolated points, from which they can be transmitted to the various " points of support " by some system of simple tensions and thrusts, meeting at these and (in general) at other points. The lines of these tensions and thrusts, if drawn to scale, give us a " skeleton diagram " corresponding to one (theoretically) permissible type of " frame "; the points in the " skeleton diagram " at which the lines of action intersect are termed " nodal points ", and represent the positions at which joints will be required in the actual framework.

It is the general problem of " Design " to devise, in the light of experience, the skeleton diagram best suited to a specified load-system. From the more restricted standpoint of stress-calculation (and of this paper), we may assume the skeleton diagram, as well as the external forces acting at the nodes, to be specified.

2.42 *" Primary " and " Secondary " Stresses*

Very little consideration is required to show that the theoretical " frame " of the foregoing paragraph is an ideal to which any actual structure can only approximate. Lines of thrust or tension, in the skeleton diagram, have to be replaced by structural members of sufficient cross-section to withstand the forces imposed upon them, and it is not to be hoped that connections sufficiently robust to transmit these forces between one member and another can be made to ensure the theoretical requirement that no bending action shall be brought to bear upon the members: except in the simplest cases, where the applied forces all act in one plane, so that " pinned " joints can be designed which restrict the bending actions within narrow limits imposed by friction, the requirement cannot be satisfied even approximately. Hence, in most cases, no attempt is made to retain it as a dominating requirement in design, and members are connected by more or less rigid gusset-plates, etc., which remove all pretence to freedom of the joints.

From this practice arises the custom of regarding the actual stresses in a framework as made up of two parts,—*primary stresses*, by which are meant those stresses which would operate if the joints of the framework were completely free; and *secondary stresses*, by which are meant the additional stresses which come into operation solely as a consequence of the fixity of the joints, and of the

* We may assume that the equilibrium of the structure is stable under any applied system of external loads.

† In the paper cited, a corresponding proof was advanced to justify the method proposed by Castigliano for use in the more general case, where the conditions are such that the initial strains have specified values.

fact that the applied forces are not in every case applied at the joints of the structure. Definitions equivalent to these were given in the report of the Airship Stressing Panel;* it was there emphasised that the term "primary" does not involve any restriction to non-redundant frameworks (§2.44).

2.43 *The Special Importance of Primary Stresses*

The terms "primary" and "secondary", thus employed in technical relation to stresses, should not be understood to contain any suggestion of relative importance;† but a little consideration will show that primary stresses have in fact a dominating influence upon the general design. Admitting the physical basis of Castigliano's Theorem, as propounded in §2.31 above, we see that the whole stress-distribution is conditioned by the general requirement, that the stored strain-energy shall have its minimum value. Hence, if two distinct systems of elastic constraint combine to limit a particular displacement, one being powerful and the other weak, the displacement will in fact attain very nearly the same magnitude as it would have had if the stiffer system had acted alone; for sensible departure from this value would involve considerable strain-energy in the stiffer system, and could involve only a slight reduction of the strain-energy involved in the weaker.

Now although, in any quadrilateral which forms part of an elastic frame, we can imagine the diagonal member removed without involving collapse of the structure, provided that the joints are stiff and the remaining members capable of resisting flexure, yet in general the resistance which will now be offered to an alteration in the length of the diagonal is very much less than what would have been offered by the diagonal member itself. It follows that the resultant tension or compression in the diagonal member will be very little affected by fixity of the joints.

The foregoing argument does not indicate that "secondary" stresses are unimportant, but *merely that they have little influence upon the magnitude of the primary stresses.* When the primary stresses are given, it is a simple matter to determine the corresponding displacements of the joints, and hence to calculate the changes which must occur in the angles of any "nodal triangle",‡ at the cost of flexure of the members and/or of yielding at the joints: the stresses thus introduced will be, in the sense of §2.42, "secondary," but it will often happen that they dominate the design of the individual members. To sum up:—it is only in relation to the general design of the complete structure that primary stresses assume primary importance for the question of strength; but it is generally true that the primary stresses control the secondary, whilst the secondary have a negligible influence upon the primary.

2.44 *"Simple" and "Redundant" Frames*

It follows incidentally from the argument of the preceding paragraph that in general it is not necessary to deal with an engineering framework as a whole, except when our concern is with primary stresses; that is to say, so long as we are dealing with complete engineering structures (as distinct from the members which compose them) we may assume complete freedom of the joints. The framework is now specified by its "skeleton diagram" (§2.41); the action in each member will be a simple tension or compression along the line which joins its end points; and the elastic properties of a member will be sufficiently defined, if we know what change of length is produced in it by a tension or compression of specified amount.

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* R. & M. 800, § 4.

† *Cf.* the Report of the Airworthiness of Airships Panel, R. & M. 970.

‡ That is, the triangle formed by joining three nodes by straight lines.

A new and important classification now suggests itself,—a framework falling into one or other of two classes, according as the loads in the individual members can be calculated from statical considerations alone, or depend upon its elastic properties. Frameworks in the first class are termed " simple ", those in the second are termed " redundant ".

2.45 *The Criterion of a " Simple " Framework**

Let j be the number of " nodal points " in the " skeleton diagram " (§2.41), —*i.e.*, of " joints " in the actual framework; and let m be the number of the " members ": then by writing down the three conditions of equilibrium for each nodal point, we shall obtain $3j$ equations relating the m unknown actions in the members. But these are not all independent; for by adding all the equations which relate to equilibrium in one particular direction, we obtain an equation which involves the external forces only, and which expresses one of the conditions of equilibrium for the framework as a whole. Since the external load-system is necessarily self-equilibrating, three of the $3j$ equations can be shown in this way to be superfluous, and three more can be seen to be superfluous in view of the consideration that the structure as a whole must have no tendency to turn about any axis in space.

In all, then, we have $3(j-2)$ independent equations derived from statical considerations; and in order that these may be sufficient to determine the action in every one of the m members, we have the relation

$$m = 3\,(j-2). \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

This relation must obtain in a simple three-dimensional framework (or " space frame "). For a framework having all its members in the same plane (a " plane frame ") the corresponding relation is easily seen to be

$$m = 2j-3. \qquad . \quad . \quad . \quad . \quad . \quad (2)$$

If the number of members, in relation to the number of joints, is less than is given by (1) or (2), the structure degenerates into a mechanism having one or more degrees of freedom: if it is greater, the structure is a redundant framework. Thus (1) and (2) are *necessary* conditions for a simple framework. They are not *sufficient*, since they could obviously be satisfied in a structure consisting of a mechanism attached to a redundant framework as base; and for this reason they should not be used uncritically: if any doubt exists as to whether a frame is simple or not, it should be studied systematically, to see whether the whole could be built up progressively from a triangle or tetrahedron (the units for a plane and space frame respectively) as foundation, in such a way that each new joint is connected to the existing framework by sufficient members (two for the plane frame: three for the space frame) to fix its position, but no more.

2.46 *" Self-Straining " Impossible in a Simple Framework. " Pseudo-Redundant " Frames*

Very little consideration is required to show that initial stresses of the kind contemplated in §2.31, arising out of internal strains in the structure (or from " self-straining ") cannot occur in a simple framework. For when the external forces are zero, the $3\,(j-2)$ independent equations of §2.45 are composed solely of terms which are linear functions of the actions in the members, with constant numerical coefficients; by (1) they are equal in number to the unknowns; and the only possible solution (if regard is paid to the remarks at the end of §2.45) is that in which every action is zero.

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* This section is based on a more detailed discussion which is given in my paper on " Primary Stress-Determination in Space Frames " (*Engineering*, February 6th, 1920).

One last type of framework remains to be noticed. It often happens (especially in aircraft structures) that a framework, although apparently redundant, includes members which are too flexible to withstand any appreciable compression, and that of these enough go out of action, under the load-system considered, to render the framework, in effect, simple. Such a framework is termed " pseudo-redundant ". It presents a problem of some intricacy, which will receive further notice below.

2.47 *Primary Stress-Determination in Redundant Frameworks*

When the number of members, in relation to the number of joints, is greater than m as given by (1), the framework is termed " redundant ". The " degree " of the redundancy is given by n, where

$$n = m - 3\ (j - 2), \qquad . \quad . \quad . \quad . \quad . \quad (3)$$

and it is clear that n additional relations will be required in order that the action in every member may be determined.

Castigliano's Method of Least Work (§2.3) provides these additional relations. For if we replace each of the n redundant members by unknown forces acting in opposite directions along its line of action, the structure becomes a simple framework loaded by external forces of which the values include n unknowns. The actions in the remaining members can be determined as linear functions of these n unknown quantities, with known constant terms; and hence the strain-energy in the whole structure (including the members classed as redundant) can be evaluated. It contains the n unknowns, and the condition that it shall have a minimum value therefore yields n equations of the type of equation (1) of §2.3. From these equations (which are linear in form) the unknown quantities can be found, and these, as we have seen, determine the action in every member of the framework.

The " Method of Comparison of Deflections " (§2.31, footnote) serves equally to supply the required additional relations, but involves the evaluation of the displacements as an intermediate step. As a second alternative (only theoretically practicable), we might work in terms of the displacements of the joints, which will be defined by $3j$ component displacements. Six of these may be specified as zero, in order to eliminate " rigid-body displacements " of the framework as a whole, and hence we have $(3j-6)$ unknown quantities, in terms of which we can specify the action in every member: the $(3j-6)$ independent equations of equilibrium (§2.45) are therefore exactly sufficient in number for a solution. In practice, however, the labour involved in this method would be prohibitive.

2.48 *Analytical Methods of Calculation for Primary Stresses in Space Frames. " Tension Coefficients," and the Quantity " Ω "*

We have seen (§2.44) that it is in general sufficient to consider primary stresses only, when dealing with a framework as a whole; and (§2.47) that Castigliano's Method of Least Work, as applied to redundant frameworks, reduces in effect to the determination of primary stresses for a " simple " framework, *plus* some additional computation involving scalar as distinct from vector quantities. It follows that the basic problem of our whole subject is primary stress-determination in simple frameworks.

In most of the problems which arise in orthodox structural engineering, it is possible to arrange, by fairly obvious devices, that attention may be confined to a plane frame. The graphical methods which have been devised for use in this special case are well known, and need not be elaborated here: the use of " reciprocal figures ", and of " Bow's notation ", is explained at an early stage in most courses of structural theory.

When, as in airship design, the structure cannot by any practicable device be reduced to an equivalent plane frame, graphical methods lose their advantages. Indeed, no general method, so far as I am aware, had been propounded for practical use before the publication in 1920 of my paper on " Primary Stress-Determination in Space Frames."*

Some of the results of that paper have already been quoted. The methods of analysis which it suggested have since been developed and employed to deduce further results, of particular application to aeronautical structures, which will be more appropriately discussed at a later stage; and as it stands, the method has been adopted for routine use by the design office of at least one engineering firm. Here it is sufficient to remark that the basic notion is the employment of a " tension-coefficient ", denoted by the symbol T_{AB}, to define the action in any member AB,—this tension-coefficient being defined as the total tension in the member, divided by its length. It is easily seen that the component in the x-direction of the force exerted on a joint A by a member AB will be given by

$$T_{AB}\ (x_B - x_A),$$

and that the similar component of the force exerted by AB on the joint B will be given by

$$T_{BA}\ (x_A - x_B).$$

Since T_{AB} and T_{BA}, by definition, are identical scalar quantities, we thus have a consistent system for expressing the forces on the joints, and signs will take care of themselves.

The conditions of equilibrium for any joint A of the framework now take the form

$$\left.\begin{aligned} T_{AB}(x_B - x_A) + T_{AC}(x_C - x_A) + \ldots + X_A = 0, \\ T_{AB}(y_B - y_A) + T_{AC}(y_C - y_A) + \ldots + Y_A = 0, \\ T_{AB}(z_B - z_A) + T_{AC}(z_C - z_A) + \ldots + Z_A = 0, \end{aligned}\right\} \quad \ldots \quad (4)$$

where X_A, Y_A, Z_A are the components, in the directions of x, y and z respectively, of the external forces applied at the joint A, whilst B, C, . . . etc., are the joints directly connected with the joint A. It will be noticed that the values of the coefficients, of type $(x_B - x_A)$, which appear in these equations can be read off at once from an ordinary engineering drawing of the skeleton diagram.

The problem of stress-determination in space frames thus reduces to the solution of simultaneous linear equations, which in a simple framework need at no time be more than three in number. In extending the analysis to problems in redundant frameworks,† where the elastic properties of the different members enter into the problem, it was found convenient to introduce an elastic constant for each member, denoted by Ω, and defined by the equation

$$P = T.l = \Omega l^3 e, \quad \ldots \quad (5)$$

P is the total tension in the member considered, T its " tension-coefficient " (as defined above), l its length, and e the fractional increase, due to strain, in the distance between its end points.‡ With this notation, the strain-energy stored in a member is given, in terms of the tension-coefficient, by the convenient relation

$$v\ [= \tfrac{1}{2}P\ (l.e) = \tfrac{1}{2}Tl^2e] = \tfrac{1}{2}T^2/\Omega, \quad \ldots \quad (6)$$

and the whole of the work involved in the " Method of Least Work " can be conducted in terms of tension-coefficients.

The application of the foregoing procedure to a representative non-redundant framework (the Forth Bridge) was described in the first of the papers

* *Loc. cit.*, § 2.45, footnote.

† R. & M. 737, § 4; R. & M. 821, § 9.

‡ For a member of uniform cross-section A, Young's Modulus E, $\Omega = EA/l^3$.

cited above: its advantages as a method of general analysis, applicable to problems in redundant frameworks, are illustrated by my paper "On Castigliano's Theorem, and the Principle of St. Venant," published in the *Philosophical Magazine* for January, 1923.*

This completes our review of available theorems and methods in the Theory of Structures. We proceed to discuss some of the special problems which are presented by the hull of a rigid airship.

3 *Primary Stress-Determination in the Hull of a Rigid Airship*

In passing to the discussion of airship problems in particular, evidently the first matter for consideration is the nature of the skeleton diagram (§2.41) which should be taken as representative of current tendencies in the design of rigid airship hulls.

3.1 *Requirements by which the Design of the Hull is Conditioned*

(*a*) The hull of course has, as its essential functions, to contain the gas from which the airship derives its lifting power, and to transmit the upward pressure of the surrounding air† to those points from which the loads are suspended. If these were the only requirements imposed upon design, the best form would be a pear-shaped solid of revolution, similar to that of the ordinary "free" balloon. But much more than this is demanded: the volume occupied by the gas (about 1,000 cu. ft. for every 68lbs. of load carried) is so large, and speed is a matter of such importance, that the shape is in fact determined almost entirely by the criterion of low aerodynamic resistance to horizontal flight; and hence, instead of the pear-shaped solid of revolution with axis vertical, we are led to the fish-shaped solid of revolution with axis horizontal,—obviously a form diametrically opposite in type to what structural considerations, taken alone, would suggest.

(*b*) The gas-container, of the external form just described, must be not only *strong*, but also (what is not necessarily the same thing) *stiff* to resist distortion under the forces which it will experience in flight. This again is an aerodynamic requirement: serious distortion of the external form would appreciably increase the resistance of the airship at high speeds, whilst flexibility of the hull structure, regarded as a long beam which moves parallel to its own length, would diminish the efficiency of the controls. It has long been known that an elongated body, opposed to a current of air, tends to set itself across, instead of along, the direction of the main current. To counteract this tendency, fins (or control surfaces of some other type) must be fitted at the after end of the hull structure, whereby the airship may be held on its course; and their action introduces aerodynamic couples at the after end, which have to be transmitted through the hull structure, in order to neutralise the unstable turning couples experienced at the forward end.

(*c*) The space available for gas must be sub-divided along its length, both to reduce the leakage which would result from accidental damage, and in order that the pressures imposed upon the hull by reason of the unequal densities of gas and air may not attain undue intensity when the ship takes up an inclination to the horizontal. Such sub-division has, of course, advantages on the structural side: it affords us some opportunity of stiffening by transverse bulkheads the long hollow structure, exposed to bending and shearing actions, which aerodynamic requirements render necessary.

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* Also published as R. & M. 821 of the Aeronautical Research Committee.

† It is a fallacy sometimes encountered in conversation, that the buoyancy of an airship is derived from the upward pressure of the hydrogen. That this is not strictly correct will be apparent when it is recognised that a greater lifting power would be obtained if the hydrogen could be replaced by a vacuum.

(*d*) The great importance of maintaining the purity of the gas as long as possible necessitates the employment of fabric containers known as " gasbags ", which are contained within the structure, but to all intents and purposes are unconnected with it. Again, the variation of atmospheric density with height makes it necessary, if gas is not to be wasted, that these bags should permit large changes in the volume of the included gas, so that a bag which is full when the airship is at its " ceiling " may contract as the airship descends. Taken in conjunction, these two requirements demand that the cells into which the hull is divided by the bulkheads (the " bays " of the airship) shall not themselves be traversed by structural members, each of which would have to pass through the gas-bag, and so would involve joints constituting a potential source of leakage.

This last restriction seriously increases the difficulties of design. A hollow circular tube is known to be inefficient as a means for transmitting shear, and structural considerations clearly indicate the desirability of running " shear wires " diagonally through the gas compartments. As it is, the shear must be taken round the outer surface of the hull.

(*e*) There is one further restriction imposed by practical requirements, which renders more difficult the design of the bulkheads mentioned above [section (*c*)]. Under normal conditions, when the airship is in level flight the pressure of the gas on the ends of a gas-bag, and hence the pressure of the bag against a bulkhead, will be neutralised by equal and opposite pressures exerted by the adjoining gas-bag : the hull as a whole will be subjected to tension by the pressures exerted by the end bags, but the bulkheads, for all practical purposes, may be said to be unstressed. When, however, the ship is " pitched " up or down, the pressures which act on opposite sides of a bulkhead will no longer be equal : the force of buoyancy, for any gas-bag, acquires a component along the axis of the ship, and this component must be transmitted to the hull structure by the bulkhead. Even in level flight, conditions of a similar kind have to be contemplated; for it is recognised that advantage should be taken of the sub-division of the gas space (as of water-tight compartments in sea-going ships) to arrange that accidental leakage from one compartment shall not involve difficulties more serious than those which attend the consequent reduction in " lift ".

A bulkhead must, therefore, be capable of sustaining pressure from either of the gas-bags which adjoin it, when the other bag is deflated; that is to say, is must be capable of resisting forces acting normal to its plane and in either direction along the axis of the ship. It may perform this function, either in virtue of its own flexural rigidity (so that the bulkheads may be regarded as stiff circular plates), or in virtue of its tension (so that the bulkheads may be regarded as flexible membranes). The first plan involves prohibitive structural weight. In the second, where the bulkhead acts as a membrane, it cannot be made flat without introducing prohibitive radial forces on the hull structure,—just as the weight of a tennis net, when the cord is strained to be approximately flat, must impose large horizontal forces on the posts. But if it is made slack, it must have freedom to bulge to either side, and hence the hull must not include any structural member running diametrically through the cross-section. Ability to resist forces which tend to " oval " the cross-section (a very important characteristic from the structural point of view) must therefore be attained by providing at every bulkhead a suitably designed *stiff ring*.

3.11 *Nature of the Skeleton Diagram*

Considerations of the kind just stated have led to a design of rigid airship hull which can be represented by the skeleton diagram shown in Fig. 2.* Only

* I am indebted to the Design Staff of the Royal Airship Works, Cardington, for kindly supplying this diagram.

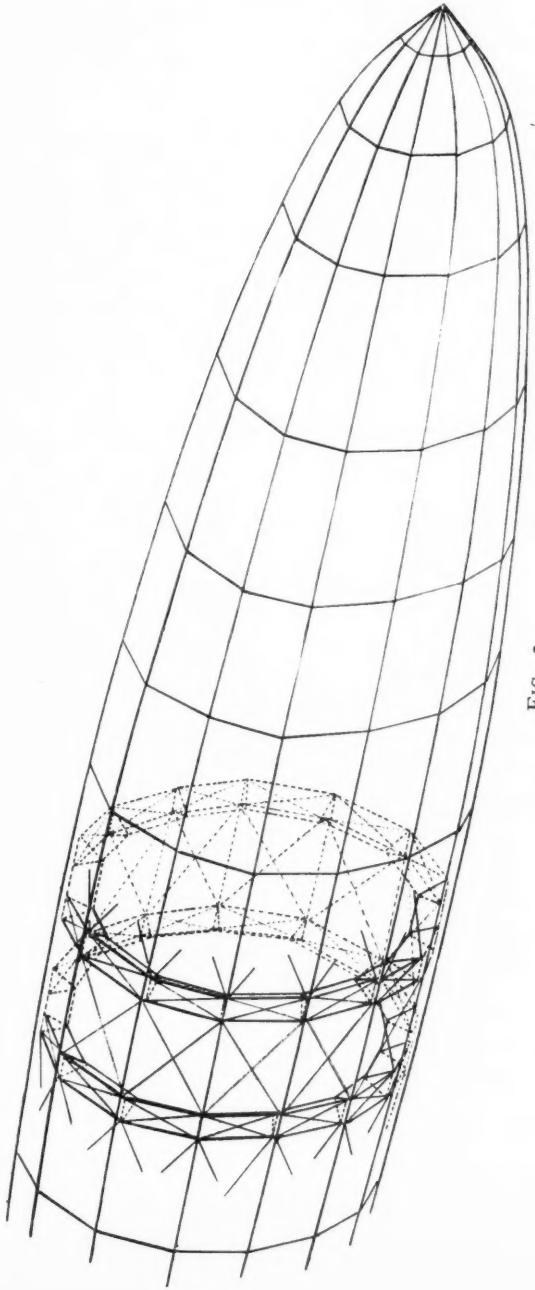

FIG. 2.

the main hull structure (the "gas container" of §3.1) is represented in this diagram; appendages such as fins, engine and passenger cars, gun mountings, etc., although individually they present important problems of stress-calculation, may in imagination be separated from the main structure and replaced by appropriate "external forces," imposed upon it at the "nodal points" (§2.41) of the skeleton diagram.

Thus simplified, the skeleton diagram may be regarded as a closed shell of more or less regular polygonal cross-section (the number of sides in the polygon being sufficiently great to render it a close approximation to the circle). It is composed of "longitudinals" (corresponding to the generators of a surface of revolution), which give it the requisite streamline form; of stiff polygonal rings (each forming a separate framed structure), which provide points of attachment for the flexible bulkheads; and of diagonal bracing members, which connect opposite corners of each of the rectangular panels formed by two adjacent longitudinals and the sides of two adjacent "stiff rings" (or "transverse frames"). In general the curvature of the longitudinals is small, and in the skeleton diagram, for the purposes of primary stress-determination, they may be taken as straight between any two consecutive joints.*

It will be recognised that bending actions, imposed upon the hull structure as a whole, are resisted in the main by the longitudinals, as also are axial tensions or compressions; shearing and twisting actions, on the other hand, are resisted mainly by the diagonal bracing system. It was at one time customary to reinforce the structure against bending by incorporating a "keel" of triangular cross-section; this involved an additional longitudinal near the bottom of the hull, either within or outside of the main shell. In those days, however, attention was focussed more or less completely upon the bending which resulted from *static* loads (*i.e.*, loads in a vertical plane): increasing recognition of the importance of *aerodynamic* loading (which may involve bending in a horizontal plane) has resulted in a tendency to suppress the keel and to revert to a symmetrical form of structure of the kind just described.

3.12 *Nomenclature*

Many of the terms which are required in a discussion of the problem of airship stressing have been defined by implication in the foregoing paragraphs. Others have still to be introduced, and it may be convenient to include at this point a complete list of definitions.

"Bay"	defined in	§§3.1, 3.2.
"Bulkheads"	,,	§3.1 (*c*) and (*e*).
"Camber"	,,	§4.23.
"Chord Members"	,,	Appendix I., §6.
"Component Actions"	,,	§3.31.
"Device of the Separate Panel" ...	,,	§3.6.
"Diagonal Bracing Members" ...	,,	§3.11.
"Frames"	,,	§2.4.
(See also "Simple, Redundant, Pseudo-Redundant" and "Space Frames")		
"Frames, Transverse" (see "Stiff Rings")		
"Generalised Formulæ"	,,	§§3.3, 3.32.

* In the actual framework, where these members are subjected to lateral (*i.e.*, "secondary") loading throughout their length, some degree of curvature may be desirable; at the bow, where the external surface of the hull must have considerable curvature, special devices are introduced to render the longitudinals effectively straight in the structural sense—*i.e.*, to arrange that they shall behave under compression as straight, and not as bowed struts.

"Helical System" of Wiring ...	defined in	Appendix II.
"Initial Tensioning"	,,	§3.4.
"Longitudinal Members"	,,	§§3.11, 4.2.
"Neglected System" (see "Residuary System")		
"Nodal Points"	,,	§2.41.
"Nodal Triangle"	,,	§2.43.
"Ovalling"	,,	§§3.1 (*e*), 3.62.
"Panels"	,,	§§3.11, 3.2, 3.23.
"Pinned Joints"	,,	§2.42.
"Primary Stresses"	,,	§2.42.
"Pseudo-Redundant Frames" ...	,,	§2.46.
"Redundant Frames"	,,	§2.45.
"Residuary System"	,,	§3.62, Appendix I., §3.
"Secondary Stresses"	,,	§§2.42, 4.
"Self-Straining"	,,	§§2.31, 2.46.
"Shell"	,,	§§3.11, 3.2, 3.62.
"Simple" Frames	,,	§2.45.
"Skeleton Diagram"	,,	§2.41.
"Space Frames"	,,	§2.48.
"Stiff Rings"	,,	§§3.11, 3.22, 3.23, 3.62, Appendix I.
"Tension-Coefficients"	,,	§2.48.
"Ω"	,,	§2.48.

3.2 *General Aspects of the Stressing Problem. The Criterion of Redundancy*

As stated at the beginning of §2.44, we may confine our attention to primary stresses (*i.e.*, we may treat the joints of the skeleton diagram as frictionless) when discussing the hull structure as a whole. But the formula (3) of §2.47 shows that, even on this assumption, the skeleton diagram is a frame which exhibits a very high degree of redundancy. Let us begin by imagining each of the stiff rings to be replaced by a freely-jointed polygon of simple bars; let N be the number of sides in the polygon and let C be the number of bays, so that there are $(C-1)$ of these freely-jointed transverse rings. Then in the structure as thus simplified we have, for the purposes of the formula quoted :—

$C \times N$ members composing the longitudinals,

and $(C-1) \times N$ members composing the freely-jointed transverse rings,

—that is, $(2C-1)\,N$ members in all, not counting the diagonal members.

. . . . . (1)

The number of the joints is N for each of the transverse rings, *plus* 2, these being the apices of the pyramids which form the end bays of the hull structure. So, in the formula (3) of §2.47,

$$j = 2 + (C-1) \times N, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and to make the frame just stiff but non-redundant ($n=0$), we should require as the total number of members

$$\begin{aligned} m &= 3\,(j-2), \\ &= 3\,(C-1)\,N. \quad . \quad . \quad . \quad . \quad . \quad (3) \end{aligned}$$

Now we have seen in (1) that $(2C-1)\,N$ members are contributed by the longitudinal and transverse members alone; hence $(C-2)\,N$ members are still required to make the structure stiff but non-redundant. But this is precisely the number of the quadrilateral panels in our framework; for of the C bays into which it is divided by the transverse rings, two are pyramidal, and the others have N panels each.

It follows that *our hollow streamline hull (with freely-jointed transverse rings) could be rendered just stiff by the insertion of one diagonal member in every quadrilateral panel.*

This result can also be deduced as a corollary of Jellett's theorem,* that any distortion of a completely closed surface necessarily involves extension of that surface; for by inserting one diagonal in each quadrilateral panel we shall make the hollow "shell" of our hull structure a completely triangulated framework, and this is precisely the condition which must be satisfied in order that resistance may be offered to the relative displacement of any two joints in that shell.

3.21 *Order of the Redundancy Given by the Crossed Diagonal Bracing*

It follows from the result just given that the insertion of two (crossing) diagonal members in each panel, each capable of withstanding end compression as well as tension, introduces a degree of redundancy given by

$$n_1 = (C-2)\,N. \qquad . \quad . \quad . \quad . \quad . \quad (4)$$

If, however, the diagonal members are incapable of sustaining compression, the structure will be rendered only "pseudo-redundant" (§2.46) from this cause.

3.22 *Order of the Redundancy Given by the Stiffness of the Transverse Rings*

Again, by making the transverse rings stiff in themselves to resist forces in their own planes, we shall in effect be introducing $(N-3)$ additional members in each ring; for each freely-jointed ring has N members and N joints, and the formula (2) of §2.45 shows that $(2N-3)$ members, in all, are required to render a plane frame of N joints "simply stiff." There are $(C-1)$ of these transverse rings, and hence, if they are all stiff, the degree of redundancy in our framework will be further increased by the amount

$$n_2 = (C-1)\,(N-3). \qquad . \quad . \quad . \quad . \quad . \quad (5)$$

3.23 *Impossibility of Reduction to an Equivalent "Simple" Framework*

From the foregoing results it follows that the skeleton diagram of Fig. 2 exhibits a degree of redundancy given by

$$n = n_1 + n_2 = (2C-3)N - 3(C-1), \quad . \quad . \quad . \quad . \quad (6)$$

if we may regard both diagonals of any panel as simultaneously effective members; or by

$$n = n_2 = (C-1)\,(N-3), \quad . \quad . \quad . \quad . \quad (7)$$

if we may treat the crossed diagonal members as "pseudo-redundant."

Remembering that C and N are each of the order of 12 or more, we see from these results that, whether (6) or (7) be the appropriate formula, it is quite out of the question to employ in routine calculations the general and (theoretically) exact methods for investigating redundant frameworks which were described in §2.47. In any such method, for dealing with a framework of redundancy n we require to solve n simultaneous linear equations with numerical coefficients, and this operation involves the evaluation of an arithmetical determinant of the nth order; it will be generally agreed by designers having experience of these methods that the labour becomes intolerable, and the attainable accuracy insufficient, when n exceeds 10 or 12.

Similar problems of excessive redundancy are presented in structural engineering of a more orthodox kind; the Forth Bridge, for example, is highly redundant if both of the diagonal members in each panel are treated as simultane-

* *Trans. Roy. Irish Acad.*, Vol. XXII (1854), pp. 343-377. A summary of this paper is given in Appendix II. of Dr. R. Jones' report "On the Aerodynamic Characteristics of Parachutes, etc." (R. & M. 862).

ously operative. The remedy usually adopted is to reduce the framework to an equivalent simple or slightly redundant frame by " intelligent " suppression of redundant members, and in this way to render the problem tractable,—allowance being made subsequently, and in approximate fashion, for the influence of the redundant members actually present. I have heard it suggested that a similar procedure might be adopted in relation to airships. But although there is much to be said for dealing in this way with the crossed diagonal bracing, since we may be fairly certain that one of each pair of diagonals will have become inoperative by the time that conditions of *failure* (which are our main concern in practice) have been attained, it does not appear that the redundancy can be reduced by this artifice below the order given by equation (7).

It is a principle of general application, that a member cannot safely be suppressed in calculation if the resulting displacement is such as would have involved a large storage of energy in that member had it been present.* Now although (as was shown in §3.2) the hull structure would still be " stiff " (in the technical sense) if the rigidity of the rings were eliminated as well as one diagonal from each quadrilateral panel, we should find that the transverse cross-sections " ovalled " to a very serious extent under the action of the upward forces imposed by gas pressure and the downward forces imposed by gravity, and this consideration shows that the flexural rigidity of the rings is too important a factor to be neglected. If, on the other hand, we retain the " stiff ring " in our calculations, the only way in which the redundancy could be reduced to a tractable figure is by suppressing other diagonal members to a number approaching the right-hand side of (7). We might, for example, suppress $(N-3)$ of the remaining diagonal members in each of the $(C-2)$ bays in which they occur. But this would leave only three diagonal members in each bay, and it is evident that the behaviour under shearing or torsional actions of the " shell " as now modified would be very far from representing the actual facts.

The conclusion seems inevitable, that the complete hull structure of a representative airship exhibits a degree of redundancy which puts orthodox methods of stress-calculation entirely out of the question; and that special methods must be devised for dealing with the structure as a whole.

3.3 *The Work of the Airship Stressing Panel. Its Search for Generalised Formulæ*

That the foregoing considerations were realised by the Airship Stressing Panel at the outset of its investigations is made clear in the introductory section of its report.† In explaining the standpoint which it had adopted in its attack on the problem of stress-calculation, the Panel gave reasons for its concentration on primary stresses, and went on to say: " In many problems of orthodox structural engineering, it is customary to neglect altogether, in the first place, sufficient members to reduce the structure to a simple frame, and straightforward methods are then available for calculating the primary stresses. The solutions thus obtained give, for any load-system, a perfectly definite expression for the action in each member, but without further investigation these expressions cannot be accepted as exact. Now a little consideration of the airship structure reveals the fact that redundancy is its most important characteristic: by neglecting counterbracing, and other features of the same kind, we shall obtain values for the primary stresses and deflections which, if they could actually occur, would impose quite prohibitive racking actions upon the joints of the framework. . . .

" Thus, although methods are available which render a three-dimensional framework, if only slightly redundant, as amenable as a plane frame to primary

* Indeed, we may go further, and assert that the member may with close approximation be replaced by a *constraint*. *Cf.* Rayleigh, *Scientific Papers*, Vol. IV, p. 451.

† R. & M. 800.

stress calculation, all possibility of treating the airship structure as a whole, and calculating the primary stresses by the methods which are normally employed, *e.g.*, for bridges, is removed by its extreme redundancy. The only hope of success was felt to lie in the discovery of generalised formulæ, or methods,—approximate only, but with more or less definite limits of error,—whereby the stress in any member of the structure can be related to the total action of the applied forces at the section in which that member lies. The energies of the Panel have accordingly been concentrated upon the discovery of such methods, either in published literature or by means of original work, and upon the testing of their accuracy by applying them to representative problems for which exact solutions could be obtained."

3.31 *The Six Component Actions in Any Bay*

As the last extract shows, the problem attacked by the Airship Stressing Panel was to discover formulæ enabling the stress in any member of any bay to be calculated in terms of the resultant action in that bay. It thus adopted an outlook similar to that which underlies the ordinary "engineers' theory" of flexure in girders; but somewhat different considerations enter into the airship problem, in that the restriction of the investigation to primary stresses implies the assumption that loads are applied only at "nodal points,"—*i.e.*, at the joints of the stiff rings. Further, it is not safe to neglect, without further investigation, any of the six component actions which make up the resultant action on any bay: shear, for example, attains an importance which it only rarely possesses in more orthodox structural design.

The six component actions referred to are:—(1) axial tension or compression, (2) shears in the vertical and horizontal planes containing the axis of the hull, (3) flexural actions in either of these planes, and (4) torsion about the axis of the hull. By the Principle of Superposition (§2.1) the stresses induced in a given member may be calculated for each component action separately and the resulting effects added.

The first point requiring emphasis is, that on the assumption (which we have just seen to be necessary) that forces are applied only at the joints of the bulkheads, the axial tension, the shears and the torsion must all have constant values throughout the length of any one bay. The flexural actions, on the other hand, will in general vary in magnitude along this length, because shear cannot be imposed on a bay unless a terminal couple is applied at the same time to maintain equilibrium: since, however, this shear is constant, the flexural couple must vary in a linear manner between one end of the bay and the other.

3.32 *Generalised Formulæ Cannot Apply Exactly in All Cases*

It is clear that formulæ of the kind desired cannot be expected to yield exact results in every case. The load entailed in a specified member by a component action of specified type does not depend solely upon the magnitude of that action; for at either end of the bay we might superpose a self-equilibrating system of external forces (*e.g.*, axial thrusts at the ends of any one diameter of the bulkhead, and balancing axial tensions at the ends of any other diameter), and such a system must alter the stress-distribution in the bay, although it leaves the resultant action unchanged. As I have remarked elsewhere, "... It is evident that a given bay could be subjected to a specified resultant action by forces applied in an unlimited number of ways, and that if an exact solution can be effected for any one distribution, this has as much claim, *à priori*, as any other to be used as a generalised formula. Further considerations must decide which of all possible formulæ is most suitable for general use."*

* R. & M. 819, p. 3.

3.33 *St. Venant's Generalised Theory of Flexure, and Its Application to Airships*

In appendices to its report, the Panel made two alternative proposals for generalised formulæ; both series were based upon analytical investigations which I had published independently, in a series of papers dealing with the determination of stresses in braced frameworks.* The reader should refer to those papers on questions of detail: the following account is restricted to general aspects.

Appendix III. (based on R. and M. 791) develops generalised formulæ for a parallel bay from the assumption that this forms part of an infinitely long braced tube, down which the action under consideration is propagated without change of type or of distribution. This is, of course, precisely the assumption adopted by St. Venant in his celebrated investigations of flexure and torsion in a uniform elastic prism.† As was to be expected from his results, it leads to the conclusion that the transverse rings remain plane under axial or flexural actions, but not, in general, under shear or torsion; they will, however, remain plane when the hull structure is of the type shown in Fig. 2,—*i.e.*, when the cross-section is a regular polygon (without keel) and the longitudinal and diagonal members are uniform.‡

The formulæ thus obtained, like those of St. Venant, derive their claim to adoption for standard use from the consideration that they represent the "least energy" distributions of stress in an isolated parallel bay: as such they are the distributions to which all particular distributions must tend, when we pass from the regions in which external forces are applied.§ But an airship hull is loaded so continuously, and its length (expressed in terms of its diameter) is so small, that this claim cannot be pressed very far: as remarked by the Airship Stressing Panel, it is doubtful whether, within the length of an actual airship structure, the stresses can ever settle down to the distributions calculated.‖

3.34 *Arguments for the Alternative Assumption, that all Distortion of the Transverse Rings may be Neglected. Resulting Formulæ*

It was, moreover, regarded by the Panel as a serious objection that no account is taken, in the solutions just described, of the influence of the bulkhead bracing. At the time of the Panel's deliberations, such bracing was usually provided by a system of radiating wires in the plane of each bulkhead; but the effect of the omission will be similar when stiffness is achieved by the device of the stiff polygonal ring: even when the design of the hull is such that the solutions involve no "warping" of a ring out of its plane, they imply that the shape of the ring, *in its own plane*, is distorted, and they take no account of the forces which such distortion would in practice call into play. From the practical standpoint, the Panel decided that equally strong claims to adoption for standard use would be presented by formulæ derived from the assumption that *transverse rings are not distorted, either in or out of their planes.*

As I had shown in a paper published as R. and M. 819, this assumption leads to simple formulæ of the kind which the Panel was seeking; unlike the assumption of St. Venant (§3.33) it is easily applied to the case of a tapered bay; and it has the important advantage that "the bay under discussion is completely isolated, and the results, in consequence, are independent of the dimensions of adjoining

* R. & M. 737, 790, 791 and 819.

† *Cf.* A. E. H. Love, *Mathematical Theory of Elasticity*, Chapters XIV, XV.

‡ *Cf.* R. & M. 800, § 12.

§ *Cf.* my paper "On Castigliano's Theorem of Least Work, and the Principle of St. Venant," *loc. cit.*, § 8.

‖ R. & M. 800, § 12.

bays."* Appendix IV. of the Panel's report developed the formulæ appropriate to a parallel bay, and Appendix VI. extended them to a bay with taper.

3.35 *Summary of the Position Reached by the Airship Stressing Panel in the Problem of Primary Stress Determination. Minor Problems*

With the derivation, from two alternative assumptions, of these alternative systems of generalised formulæ, the Airship Stressing Panel brought to a close its discussion of primary stress calculation in the hull structure as a whole; and without attempting to make a definite recommendation in favour of either system, it passed on to consider various problems of detail which appeared, in the existing state of knowledge, to be important and in need of elucidation. In §15 of the introductory section of its report, it stated that "in the view of the Panel, the investigation of primary stresses has reached a stage at which it should wait until experience has shown the practical utility of the progress made."

Of the minor problems treated by the Panel in Appendices VIII. to XIV. of its report,† little need be said here. They were expressly presented "less as an argument for the adoption of definite methods than as a collection of material which the Panel hopes may be useful," and it is not surprising that some have been rendered obsolete by innovations made subsequently, in the course of the new designs. We proceed to review its work on the major problem of primary stress-calculation, in the light of experience gained in the last two years.

3.4 *Critical Review of the Airship Stressing Panel's Report. The Question of "Initial Tensioning"*

It was always contemplated by the Panel that trial of its methods and recommendations in relation to some actual design would indicate directions in which modifications and additions are required. This expectation has been realised: constant use of its report at Cardington in the last two years has revealed no serious error, but certain lacunæ have been detected in its treatment of the general problem of primary stress-determination.

In the first place, either of its alternative sets of formulæ (see §§3·33-3·34) involves the assumption that both of the two diagonal members in any panel can resist compression: when these diagonal members are flexible wires, the assumption will not be realised unless they are given an initial tension more than sufficient to neutralise the compressive actions which are imposed upon them by the external loads. The Panel recognised this fact,‡ and the advantage, in respect of stiffness, which accrues from initial tensioning: in §6 of its report it stated that "one of the most important recommendations which the Panel desires to make is that careful attention should be given to the initial tensioning of counter-bracing, etc., in order to take advantage of the fact that an initially tensioned wire can function effectively as a strut, in resisting compression imposed by applied forces."§ But, except incidentally in an Appendix,‖ it omitted to insist on the point that the stress-system brought into existence by this initial tensioning will have to be separately determined and added to the stresses calculated from its formulæ; it provided no additional formulæ for this determination; and it failed to lay emphasis on the important consideration, that *under conditions in which the hull structure is on the point of failure, one*

* R. & M. 819, p. 4.

† Appendix VII contained a detailed account, illustrated by examples, of the "Method of Least Work" (§ 2.3).

‡ *Cf.* R. & M. 800, p. 52.

§ *Ibid.*, p. 7.

‖ *Ibid.*, p. 52.

diagonal member in each panel (if the bracing is of the crossed-wire type) will certainly have gone slack.

Incidentally, this last consideration shows that initial tensioning will have no tendency to accelerate failure of the structure, and there is no doubt that the Panel's recommendation in favour of initial tensioning is in theory completely sound, since this must reduce the distortion of the hull under forces imposed upon it in normal flight. But apart from the practical difficulty of applying initial tensions of definite amount, the fact remains that, under the conditions with which we are specially concerned in stressing, most of the redundancy contemplated in the Panel's formulæ will have disappeared.

3.41 *The Alternative Formulæ*

It might be inferred from the foregoing remarks that the Panel was ill advised to work out formulæ for the main hull structure from the assumption that every diagonal member is operative. This criticism, however, cannot be maintained, because *it is impossible to say in advance which diagonal member will go slack in any particular panel.* It is thus essential that allowance for the slackening of bracing wires be made in the final stages of stress-calculation, and formulæ of the kind which the Panel worked out are the only logical preliminary.

The Panel's omission to discriminate between the relative merits of its two alternative formulæ was, as we have seen, deliberate. It contemplated (§17 of its report) that much of the discrepancy between them would be eliminated by precisely such modifications of design as have in fact taken place. From the practical standpoint, the advantages which have been shown in §3.34 to be possessed by the formulæ of its Appendices IV. and VI. are overwhelming; I shall, moreover, attempt to show that the discrepancy is in any case a matter of negligible importance, when we come to consider the stresses which obtain at the instant of failure.

3.5 *Recapitulation. The Outstanding Problem*

In this essay I have sought to justify a standpoint, in regard to the problem of primary stress-calculation, which may be summarised in the following statements:—

(i) It is not to be expected that exact methods can be devised, or that they would not be intolerably laborious if they existed. Some assistance must be brought in by reasoning of an "engineering" rather than of a rigorous mathematical nature.

(ii) Whether or no the diagonal members be assumed capable of withstanding compression, the redundancy of an airship hull structure is such as to necessitate the use of generalised formulæ.

(iii) When an airship is stressed under loads multiplied by the appropriate factor of safety (so that the conditions of stressing are such as will bring the hull up to its breaking point) the apparent redundancy of the crossed-wire bracing will certainly have disappeared. But we cannot eliminate the "pseudo-redundancy" *à priori*, because we cannot foresee which of two wires will go slack under a complicated load-system; and hence the formulæ of R. and M. 800 are the only logical preliminary in stressing.

How, if we maintain generality of treatment by the assumption of complete redundancy, can we subsequently correct our results to allow for the slacking off of one wire in every panel? This, in recent years, has come to be recognised as the outstanding problem in airship stress-calculation; it was pointed out by Prof. Pippard, in a note communicated to the Aeronautical Research Committee* as early as July, 1924.

* T. 1946 (unpublished).

3.6 *Recent Developments. The "Device of the Separate Panel"*

In a reply to Prof. Pippard's note, written in October, 1924,* I endeavoured to meet this difficulty. I suggested that there is no need to anticipate any serious degree of uncertainty as to the magnitude of the stresses in a hull of which the cross-bracing wires are initially tensioned, and that the precise tensions to which the wires are strained is not a question of any great importance. ". . . *The purpose of initial tensioning is to give the structure greater rigidity against normal loads, not greater strength to resist extreme loads;* for, as Professor Pippard has pointed out, one wire in each pair will almost certainly go slack before the structure fails, and hence the magnitude of its initial tension makes no difference to the final result."

I agreed that it was imperative to allow for this slacking off of one wire in each panel,—". . . an aspect of the stressing problem which does not appear to have been made clear in R. and M. 800"; and for that purpose I proposed what has come to be known as "the device of the separate panel." In my note I considered a rectangular panel, as shown in the following diagram:—

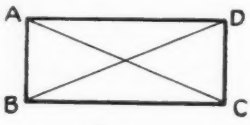

FIG. 3.

". . . Let us suppose that the methods of R. and M. 800 give, for some standard system of loading *which has been multiplied by the appropriate factor of safety,* a compression T in the diagonal member BD of the panel shown in the diagram above. It makes no sensible difference to the problem whether or no a tension was imposed originally; the point is that the compression T cannot in fact be exerted. We could, theoretically, start again and treat the structure as a frame in which this member does not appear; but this is not really practicable, and I suggest that the best procedure is to allow for the non-existence of the compression in BD by the simple procedure of imposing forces of magnitude T, acting along BD and tending to make these joints approach one another, *and calculating their effects as though these were confined to the panel ABCD.* This simplifying assumption will lead to an over-estimate of the stresses in the panel ABCD,† and to that extent will err on the safe side. It will err in the opposite direction as regards the stresses in the panels surrounding ABCD; but I think it is reasonable to assume that any 'unsafe error' thus introduced will be neutralised by the 'safe errors' involved when these surrounding panels are similarly treated in their turn."

In a subsequent note, written in March, 1925,‡ I suggested that a test of this device might usefully form one item in the programme of experimental research which was being conducted by Prof. Pippard. I also directed attention to a further consideration: ". . . It seems to me probable that the treatment suggested will lead to sufficiently accurate results when it has been carried out throughout the whole of the hull structure, even though it would give insufficiently accurate results as applied to a single panel. That is to say, the effect of removing one diagonal wire from a single panel ABCD will probably be felt throughout a much greater part of the remaining structure, when the panels in this remaining structure are still truly redundant, than when every panel has had one diagonal bracing removed."

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* T. 1946A (unpublished).

† Because the effect is in actual fact shared between this panel and the remainder of the hull structure.

‡ Appendix to T. 2036 (unpublished).

The point of this last contention will be made clear if we imagine the device applied to a redundant structure for which an *exact* solution has been obtained. For the reasons stated, errors must be introduced by the assumption in the early stages, when only a part of the redundancy has been eliminated in this way: but when the redundancy has been completely eliminated, we are left with a solution which satisfies the conditions of equilibrium at every joint; and this must be the correct solution, because the solution which satisfies these conditions in a simple framework is unique (§2.46).

Prof. Pippard proceeded* to carry out this proposed test of the method in two ways: first, analytically, by comparing the results of calculations made for a certain framework, (*a*) by the "device of the separate panel" applied to exact calculations for the completely redundant structure, and (*b*) on the initial assumption that all counter-bracing members have been removed; secondly, by experiment, applying the device to existing experimental results for a certain redundant structure, and comparing the estimates so found with fresh experimental determinations made after the counter-bracing members had been removed. Good agreement was obtained in each instance, and Prof. Pippard's conclusion is that "the method . . . is sufficiently reliable to give a good indication of the stresses in the extreme condition when redundant members have ceased to operate."†

Thus the "outstanding problem" of §3.5 appears, for practical purposes, to have been solved. All that remains is to show the nature of the correction to which we are led by the "device of the separate panel."

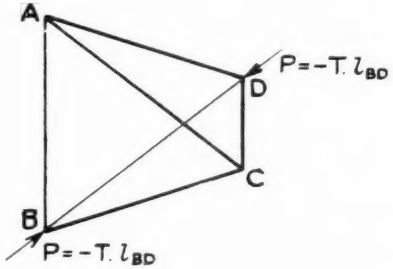

FIG. 4.

The correction does not become appreciably more complicated when, instead of the rectangular panel discussed above, we consider a trapezoidal panel, as shown in Fig. 4; hence we can apply it without difficulty to tapered bays. We suppose that the formulæ of R. and M. 800 have indicated the existence of a thrust P in the diagonal member BD, *beyond what this member can sustain.* (If, as in modern airship construction, the member BD is a wire, it cannot sustain any thrust at all; but for greater generality we may assume that it is a long strut, buckling in the manner of Euler under some definite critical thrust). We have to allow for the non-existence of a thrust P in BD; and such allowance will clearly be made if we superpose on our existing solution the effects produced by equal and opposite forces of magnitude P, acting as shown to compress the diagonal BD. *Our assumption is that these effects are confined to the members composing the panel considered.*

In the notation of §2.48, we write T for the tension-coefficient of BD

* R. & M. 971 (May, 1925). *Cf.* also Prof. Pippard's paper read to the R.Ae.Soc. on January 7th, 1926.

† *Loc. cit.*

corresponding to the thrust P. Under the convention that positive tension-coefficients relate to tensions, we have

$$T.\, l_{BD} = -P, \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

whence T is known; and it is easily verified that the effects of the applied forces P are given by

$$\left.\begin{array}{r} \text{tension-coefficients } T \text{ in AD, BC,} \\ \text{a tension-coefficient } -T \text{ in AC,} \\ \text{a tension-coefficient } T.\, l_{CD}/l_{AB} \text{ in AB,} \\ \text{and a tension-coefficient } T.\, l_{AB}/l_{CD} \text{ in DC.} \end{array}\right\} \quad . \quad . \quad . \quad (2)$$

Since T is negative, by (1), we see from (2) that AB, BC, CD, DA are subjected to addition compression, and AC to additional tension. Additional effects will of course come on the side members from similar corrections made for adjoining panels.

As already stated, corrections by "the device of the separate panel" must always be made as a final adjustment, after the effects of the different component actions have all been superposed.

3.61 *Concentration of Attention on the Conditions of Ultimate Failure*

It will be appreciated that the innovation just described involves a very important change in our attitude towards the whole problem of primary stress-calculation. In R. and M. 800, realisation of our inability to foretell which of any pair of diagonal members will go slack under a complicated distribution of loads led to treatment in which an extreme degree of redundancy was presumed: the results, in consequence, are valid only to the extent that this redundancy can be attained by initial tensioning, and they are subject to the uncertainty which must exist as to the actual magnitude of these initial tensions in practice; in any event, the redundancy will have disappeared by the time that the structure is on the point of failure. In contrast with this position, the "device of the separate panel" gives us exactly the power we require of eliminating pseudo-redundancy *a posteriori*; its use is justified for practical purposes by the arguments of §3.6, provided it is carried far enough; and if we apply it systematically, to every panel in the outer "shell" of the hull structure, we shall be left at the finish with a solution which relates to a framework of vastly reduced redundancy, which for that reason is subject to much less uncertainty, and which reproduces the actual conditions at failure. The formulæ of R. and M. 800 retain their full value, since they give us the preliminary system we require; and the uncertainty in regard to the relative merits of the two alternative sets of formulæ loses its importance in view of the change in procedure now rendered possible. This we proceed to consider.

3.62 *Relative Functions of the "Shell" and of the Stiff Transverse Rings*

Even after employing the "device of the separate panel" to the utmost, we shall be left with a redundant framework. The "shell" alone, with only one bracing wire in each panel, has been seen to constitute a "simple" frame, and in the actual ship it is reinforced by a number of stiff rings.

But at this stage we may introduce an assumption in regard to the partition of load between the "shell" and the rings, for which considerable justification can be adduced. We have seen in §§2.43 and 3.23, as a deduction from the Principle of Least Work, that when two paths are offered to the stress, it goes mainly through the stiffer path; we have also seen (§3.23) that the function of the transverse rings is to prevent "ovalling," to which the shell, though "stiff" in the technical sense, offers only slight resistance; and it is clear that the members composing the ring will not be stressed by axial load or flexure, except to a very

slight extent by interaction from the extension or contraction of the external ring members. Let us then assume, in the first place, that *the six component actions of* §3.31 (*axial load, flexural couples, torsion and shears*) *are taken by the shell members only,* and that the stresses in these members are calculable by the formulæ of Appendices IV. or VI. of R. and M. 800.

It has been emphasised that these formulæ enable us to deal with each bay as an isolated unit, and that they will give exact results as applied to any specified bay, provided that the component actions are distributed in certain definite ways. In general, the actual loads will have a different distribution: if then we consider the equilibrium of every joint, under the action of (*a*) the actual imposed loads and (*b*) the actions in adjoining members as calculated by the formulæ of R. and M. 800, we shall be left with a residuary system of unbalanced loads at each ring. In the nature of the case, this residuary system will be self-equilibrating; its tendency will be to cause distortion of the transverse ring both in and out of its plane. Our second assumption (justified by the considerations stated above) is that *the residuary system, in so far as forces in the plane of the ring are concerned, is resisted by the stiff transverse ring alone.* That is to say, we neglect altogether any resistance to "ovalling" which may be possessed by the shell.

As regards the forces which tend to cause warping of the ring out of its plane, it is difficult to make general recommendations. The further assumption that these longitudinal forces of the residuary system are also resisted by the ring alone would greatly over-estimate the "duty" required of the ring, because the shell must play the greater part in resisting such forces. For the moment we must leave this problem to be settled by practical experience in the design office: my own anticipation is, that the forces will not be found to be of sufficient importance to call for systematic treatment.

3.7 *Outline of Procedure in Primary Stress-Calculation*

This essay does not purport to be a manual of stressing, and in the present section I shall merely indicate the general bearing of its conclusions: it must be emphasised that practical considerations will modify the procedure suggested.

As explained in the introductory section, the external load system, whether due to gravity or to aerodynamic forces, is regarded as specified and not open to question; it must, of course, be self-equilibrating as regards the complete hull structure,—inertia effects, if any, being taken into account as "reversed effective forces." The first step in the process of stress-calculation will be to distribute the external loads upon the joints of the skeleton diagram (since we are concerned only with primary stresses), and thence to construct diagrams showing the variation from bay to bay of each component action. For reasons explained in §3.31, the axial tensions, the shears and the torsional couple will be represented by stepped diagrams, and the flexural couples, in general, by polygonal diagrams.

Diagrams of this kind will have to be constructed for every condition of flight which it is thought may be serious in its effects on any bay. From an examination of the complete series, it will be a simple matter to decide the crucial conditions for any given bay. Here the advantages of isolation possessed by the formulæ of R. and M. 800, Appendices IV. and VI. (*cf.* §3.34), are at once apparent: they enable us, in effect, to stress each bay independently.

By means of these formulæ, the tension coefficients for the longitudinal and diagonal members can be determined. Knowing these, and the actual forces acting on the joints of the adjoining transverse rings, we can deduce, as explained in §3.62, the "residuary system" of loads which is imposed upon each ring; and the new problem now presents itself, of stressing the stiff polygonal ring under forces in its plane. Some discussion of this problem is given in an Appendix to this essay (see §4.1).

The last stage is to introduce the corrections entailed by the "device of the separate panel." We are left with results which give the maximum tension-coefficient to be expected in every member of the hull. The problem of primary stress-calculation is thus completely dealt with, and the design of the individual members can proceed.

4 *Secondary Stresses, and the Design of Components*

Much, of course, remains to be done even after the primary stresses have been determined: fins, power cars, and other attachments have to be suitably proportioned, whilst the detailed design of the girders calls for careful attention. Some of the problems thus presented may be treated by slight and obvious extensions of the orthodox methods which have been reviewed in §2; others have been solved by means of special devices due to the design staff at Cardington, and I should be trespassing on the ground of others if I were to describe them here: there remain, however, certain problems in which I have been able to make suggestions of general method, and these may be briefly reviewed.

4.1 *Stress-Calculation in the Stiff Polygonal Ring*

Under the action of forces in its plane, the stiff polygonal ring may be regarded as a plane framework, although its section is in fact triangular. As a problem in stress-calculation, in a word, it is two-dimensional, and the orthodox methods of §2 are applicable.

The details of stress-calculation for these rings were considered by me in a note, written in October, 1924,* which is subjoined to this essay as an Appendix. Three possible types of construction were contemplated. In the first the ring may be regarded as a continuous circular girder; in the second and third, as a braced framework in which the redundancy is found to be of order 3.

No serious difficulty arises in the treatment of this two-dimensional problem. In the actual airship a ring will also be subjected to forces which tend to distort it out of its plane, and for these a method of treatment which employs the notation of tension-coefficients, etc., described in §2.48, has been found most convenient. The details have been worked out by Col. Richmond and his staff at Cardington, to whom it must be left to describe them in a future paper.

4.2 *Secondary Stresses in the Longitudinal Girders*

For the purposes of primary stress-calculation, in which the loads are assumed to act only at the "nodal points" of the skeleton diagram, the longitudinal members may be regarded as connected to the transverse rings by ball joints, and therefore as subjected merely to simple tension or thrust. They will in fact be very largely continuous, and moreover they will have to act in a secondary capacity, as laterally-loaded girders transmitting pressures from the outer cover of the airship, and (possibly) from the gasbags, to the joints of the hull structure; further, they will have to conform to the general curvature of the "streamline" hull, and it will sometimes be convenient to give them camber in the structural sense also, so that end thrust and lateral load may neutralise one another to some extent. Their detailed design is thus a problem of great importance and considerable intricacy, since the loading to which they are subjected is a three-dimensional system, and since deflections, reacting with the end thrust, will modify the internal stress-system to an important extent. The following notes on the calculation of stresses and deflections were prepared by me in 1925, in relation to a girder of triangular cross-section, with rectangular side panels and pseudo-redundant diagonal bracing.

* T. 1967 (unpublished).

4.21 *Determination of the Stresses*

Fig. 5 represents the cross-section of a straight airship girder at one of its triangular bulkheads. Any force applied at a joint such as A may be resolved into three components, one acting along the boom A, and the other two (in the plane of the paper) acting along the lines AB, AC, as indicated in the diagram.

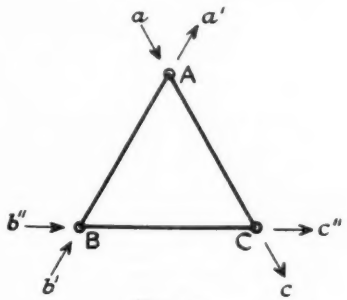

FIG. 5.

It is easily seen, from the conditions for equilibrium of the joint A in the plane of the paper, that the internal stresses in the panel AC must balance the component force a which acts in the plane of this panel; and similarly, that the internal stresses in the panel AB must balance the component force a'. Hence, provided that the girder is non-redundant (so that the stresses in the different panels are not modified by interaction through the elasticity of the booms), we may treat each panel as a separate plane framework, and determine the loads in the individual members by any of the methods which are customarily employed. In this procedure each boom will be considered twice, since it occurs as a component member of two different panels: the actual load in it will evidently be the resultant of the loads which these panels contribute severally. It is perhaps desirable to add that a component of external force which acts *along* the boom may be taken as an external load on either one of the panels, but not on both, since it would then, in effect, be counted twice.

4.22 *Determination of the Deflections*

Treating the panels separately in this way, it is an easy matter to deal with crossed-wire (" pseudo-redundant ") bracing, retaining in each bay only that one of the two wires which can be seen to be in tension under the applied load-system. The actual determination of the stresses may be effected by graphical methods, and this procedure has the advantage that the work is very easily checked. We arrive, finally, at a complete solution for the loads in the component members of the girder, and we can proceed to investigate the resulting deflections.

Here, too, we may deal with each panel as a separate plane framework, with the advantage that graphical methods can again be employed. The construction of deflection diagrams is a fairly well-known operation, which has been fully described by W. S. Farren in §6 of R. and M. 769.* In the present connection it is only necessary to add that the calculated extensions of the booms (which constitute part of the data for the deflection diagram) must be those corresponding to the *total* loads which occur in these members (*cf.* §4.21).

It remains to apply the results for the several panels to determine the

* " The Calculation of Stresses in a Redundant Structure by the Method of Comparison of Deflections, etc.," October, 1921.

distortion of the complete girder. In general, the relative motion of two different triangular bulkheads will be made up of relative tilting, due to flexure, and also of relative distortion and rotation in their own planes, due to torsion and to shear. The tilting effect is immediately calculable from the boom extensions, so that we may confine our attention to displacements in the plane of the bulkheads.

From the deflection diagram for a panel we can deduce the component deflection of any joint in a direction parallel to the bulkheads (*i.e.*, to the transverse struts). Realising that the panel, being a plane framework, will oppose no resistance to distortion out of its plane, we may proceed to construct a diagram (Fig. 6) which will give the twist and deflection of the girder. For this purpose we take the bulkheads as parallel to the plane of the paper, and we eliminate translation and rotation of the girder as a whole by fixing the joint A and the direction of the strut AB in one of the terminal bulkheads. Then from the deflection diagrams for the different panels we can find the extensions of AB, BC, CA. The direction AB is maintained, so we can draw a vector Ob to represent the displacement of B. The motion of C will consist of a known displacement bc_1, relative to B, in a direction parallel to BC, combined with an unknown perpendicular displacement due to warping of the panel BC. Equally it must consist of a known displacement Oc_2, relative to A, in a direction parallel to AC, combined with an unknown perpendicular displacement due to warping of the panel AC. These two conditions, taken together, show that the resultant displacement must be given by the vector Oc of Fig. 6.

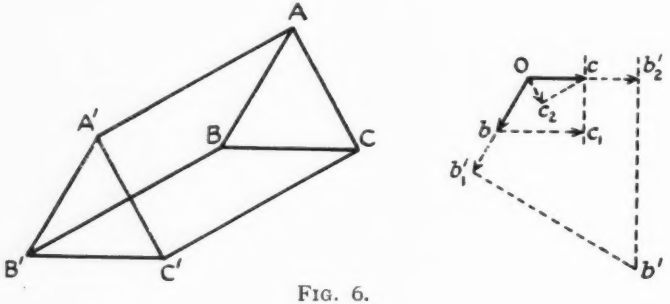

FIG. 6.

The displacements of the joints A′, B′, C′, of the adjoining bulkhead may be similarly determined. That of B′ must consist of a known displacement Ob_1', relative to the fixed point A and parallel to AB, which can be found from the deflection diagram for the panel AB, combined with an unknown warping displacement perpendicular to AB. It must also consist of a known displacement cb_2', relative to C and parallel to BC, combined with another warping displacement perpendicular to BC. Hence it is given by the vector Ob'. The procedure can be continued indefinitely.

4.23 *Girders with Camber*

The methods outlined above should prove satisfactory in actual use, and will form a useful alternative to those of R. and M. 800, especially in the consideration of girders loaded up to their limits of strength, when the redundancy afforded by crossed-wire bracing will certainly have disappeared. There is only one consideration outstanding: airship girders will generally be cambered in conformity with the general curvature of the ship, and the bending moments which result from the camber must be taken into account.

For this purpose I suggest the following artifice. Let OO_5 (Fig. 7) be the centre-line of the cambered girder, O_1, O_2 . . . being the centres of the intermediate bulkheads. By introducing suitable transverse forces V_0, V_1, . . . etc., in combination with the given end thrust P, the line of the resultant thrust may be made to pass through O, O_1, etc. . . . ; and we thus arrive at conditions which may be investigated with sufficient accuracy by treating a straight girder centrally loaded. Conversely, the conditions for the cambered girder subjected to end thrust

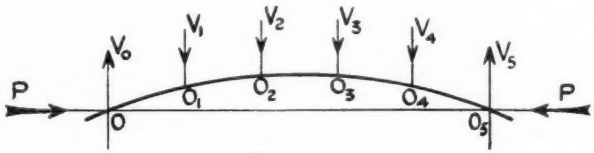

FIG. 7.

P alone may be investigated on the straight girder, provided that transverse forces equal to V_0, V_1, V_2, etc. . . ., *but opposite in sign*, are introduced as part of the external load system.

5 *List of Original Published Papers to which Reference has been made in this Essay*

"Primary Stress-Determination in Space Frames."—*Engineering*, February 6, 1920.

"On the Determination of the Stresses in Braced Frameworks." (R. and M. of the Aeronautical Research Committee.)

Part I. (R. and M. 737)—March, 1921.
Part II. (R. and M. 790)—February, 1922.
Part III. (R. and M. 791)—April, 1922.
Part IV. (R. and M. 819)—September, 1922.

"On Castigliano's Theorem of Least Work, and the Principle of St. Venant," —*Phil. Mag.*, January, 1923; also published as R. and M. 821, May, 1922.

(Unpublished papers have been subjoined to the essay as Appendices.)

APPENDICES

APPENDIX I.

On the Calculation of Stresses and Deflections in the Transverse Frames of an Airship

(*Unpublished paper No. T.1967 (October, 1924) of the Aeronautical Research Committee.*)

Introductory

1. The energies of the Airship Stressing Panel were concentrated in the main upon the problem of determining the "primary stresses" in an airship hull due to any specified load-system. For this purpose, as explained in §4 of its report (R. and M. 800), the imposed forces must be reduced to an equivalent system of loads acting at the joints of the skeleton structure, and the joints must be assumed to be perfectly free; the action in every member is then a simple tension or thrust along the line which joins its end points.

2. In view of the extreme redundancy exhibited by an airship structure, which precludes the possibility of treating it as a whole by the methods usually applied to frameworks, the Panel proceeded to develop formulæ giving the action in any member in terms of the resultant action exerted by the external forces upon some transverse section of the airship. This resultant action is conveniently represented by three component forces acting at the centre of the section, together with three component couples; and although these six actions are obviously not sufficient to define the primary stress-system exactly, the Panel found reasons for believing that no differences such as are likely to obtain in practice among load-systems which are "statically equipollent"* will involve serious departure of the primary stresses from their calculated values.

The argument may be presented as follows:—If we consider any isolated bay of an airship (*i.e.*, a portion comprising two adjacent bulkheads, or transverse frames, and the longitudinal and bracing members which connect them), the formulæ of R. and M. 800 give values for the actions in the different members which are consistent among themselves, and which would actually obtain if the bay were subjected to *some* definite system of applied forces having the resultant action specified. It follows that the errors introduced in any given instance by the use of these formulæ may be considered as resulting from the neglect of a certain load-system, which is the difference between the load-system actually imposed and the load-system implied in our formulæ. This neglected load-system will consist of a number of forces at either bulkhead which, by what has been said, must be self-equilibrating: the argument of R. and M. 800 is that it will impose negligible loads in any member other than the circumferential and bracing members of the transverse frames.

3. It must be admitted that load-systems could easily be imagined in regard to which the above argument would not be acceptable: Fig. 1 illustrates diagrammatically a type of "neglected load-system," applied at four points on

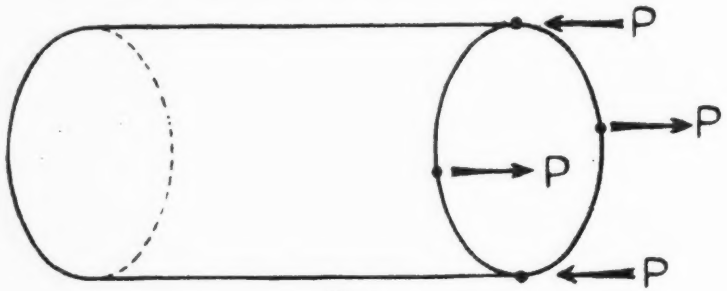

FIG. 1.

one of the bulkheads, which, although it satisfies the condition of being self-equilibrating at either bulkhead, might induce very serious actions in some of the longitudinal and bracing members. But it is difficult to see how a system of this nature could originate in practice, unless as a result of attaching engine cars in such a way that the airscrew thrust is very locally applied. Experience in actual design may be expected to reveal before long whether the conclusion is admissible, but for the present it seems reasonable to adopt the standpoint of R. and M. 800.

4. It was recognised by the Airship Stressing Panel that the "neglected load-system" includes, on the other hand, *forces in the plane of each bulkhead*

* The term employed by Saint Venant in the enunciation of his well-known "principle." *Cf.* A. E. H. Love, *op. cit.*, §80.

which in general will demand careful consideration. In §11 of its report it stated that the function of the bulkhead bracing in practice was assumed to be " that of distributing the applied forces between the different joints of the bulkheads in such a manner as to make the actual loading on the tubular structure conform closely to what is postulated in the solutions found for the braced tube "; and in Appendix XI. it made a preliminary investigation of the comparative rigidities of various forms of transverse wire bracing. But it did not discuss the general problem of stressing the bulkhead, as a plane frame, against the coplanar forces of the " neglected load-system "; and since this problem must arise at an early stage in any practical design, I have endeavoured to deal with some aspects of it in this note.

Some Possible Types of Construction

5. Two functions are imposed upon the transverse frame of an airship: in the first place it must contribute to the stiffness of the complete hull structure, principally by resisting, in virtue of its own stiffness, any tendency of forces in its own plane (arising from the " neglected load-system ") to distort it in that plane; and in the second place it has to serve as a bulkhead, transmitting to the joints of the hull framework the axial thrust which is exerted upon it by a neighbouring gasbag, either during inclined flight or when the gasbag on its other side is deflated.

To meet the first requirement it is necessary to arrange that the transverse frame shall be, in effect, a stiff ring. When this has been done, the bulkhead may be constructed as a separate component (*e.g.*, of netting) and attached. But the two requirements are not entirely independent of one another as problems in design; for unless the bulkhead is allowed to bulge under the pressure of the gasbags, or possesses flexural rigidity in itself (which would entail prohibitive weight), it must impose very heavy inward thrusts upon the stiff ring in transmitting the gasbag thrust to the hull structure. Now the bulkhead cannot bulge if it bears upon taut bracing wires crossing the ring from side to side: it will merely transmit the thrust to those wires, which will thus in effect become the bulkhead, and will be subjected to very great additional tensions, thereby imposing correspondingly heavy thrusts upon the ring members. So the present tendency of design is towards a separation of the functions of bulkhead and stiff ring, and, *as a necessary corollary, towards a type of ring which does not depend for its stiffness upon radial bracing members.* From the point of view of the ring, as was shown by the calculations of Appendix XI. of R. and M. 800, the radial system of bracing is desirable on the score of the great rigidity which it supplies for a given weight of added metal; but in practice the gasbag thrust prevents it from functioning with the efficiency which was contemplated in those calculations.

6. The ring must of course have considerable depth in a radial direction, if it is to be stiff to resist distortion in its own plane, and the bulkhead for transmitting gasbag thrust must be designed accordingly. Three distinct systems of ring design suggest themselves for consideration, and are illustrated in Fig. 2; it would seem that any design which is likely to be adopted in practice will conform to one or other of these systems. System A is in effect a continuous circular girder, for which, just as for the straight plate-girders of bridges, methods of stressing must be devised which do not involve its reduction to an equivalent framework; it would appear quite possible that this type of design will lend itself to construction in thin sheet metal such as is employed for aeroplane spars. System B bears to system A the same relation that a bridge truss of " Warren girder " type bears to a plate girder: the flanges have become separate members and the web has been replaced by a system of bracing members. System C is of a more distinct type: only the outer flange of A is retained,

and this flange is turned into a continuous series of "bow-string girders" by means of chord members which replace both the inner flange and the web or bracing members,—stiffness of the ring as a whole being achieved by arranging that the chord-members overlap. By analogy with the system adopted in the construction of wire wheels, where the spokes are too slender to act as struts, it would seem probable that the chord-members might be of a design suited to withstand tension only, so that most of the metal will be contained in the outer (rim) members, where of course it is best placed for stiffening the hull structure generally.

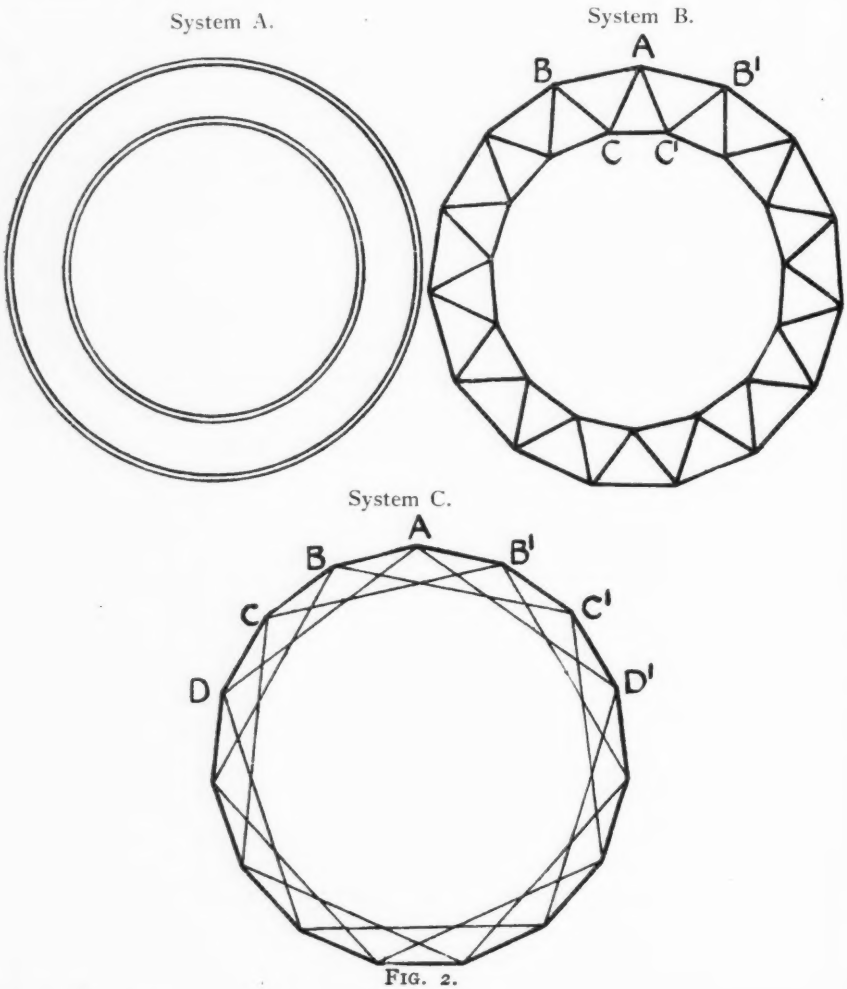

FIG. 2.

Calculation of Stresses and Deflections

7. *System A.*—If this type of construction should be developed in the future, the designer will find in existing literature on curved beams methods and formulæ

which should be sufficiently exact, and also general, to meet his requirements. Filon* has recently tested, by comparison with the results of exact calculations, the accuracy attainable by the use of the ordinary "engineer's theory" in a particular case—that of a ring subjected to tension by two equal and opposite forces applied to the internal boundary: he concludes that "the engineering approximation is quite good, provided the ring is narrow."†

8. In Fig. 2 the ring is shown, diagrammatically, as circular, in order to emphasise the fact that in this type of construction, unlike the other two, no structural advantage is possessed by the polygonal form. If means could be devised for maintaining a sufficient pressure in the space between gasbags and outer cover to overcome the aerodynamic pressures on the latter, the hull form could be made a solid of revolution, with the possibility of some aerodynamic advantage; and in this event a circular form for the ring would be preferable.

9. In any case the procedure for stress-calculation is the same. We imagine the ring to be cut through at some convenient section, and the action across that section to be replaced by a thrust along the centre line of the ring, a shear (this will of course be zero if the section is on a line of symmetry), and a flexural couple. In terms of these three unknowns, and with the load-system specified, the bending-moment at any point on the centre line of the ring (whether circular or polygonal) can be easily calculated; and it is then a simple matter, as will now be shown, to determine the unknown quantities from relations which express the fact that continuity is actually preserved at the section which was imagined to be cut through.

The bending formula involved is

$$EI\delta\psi = M.\delta s \qquad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\delta\psi$ is the change produced by flexure in the relative inclination of two sections separated by a length δs of the centre-line of the ring,
M is the bending-moment, and
EI the flexural rigidity of the ring, at the section considered.

Then if ABCA′ (Fig. 3) be the centre-line of the ring, in which the cut is supposed to be made at Section A, the relative inclination of the ends AA′ of the strained centre-line will be given by

$$\int_{A}^{A'} d\psi = \int_{A}^{A'} (M/EI)\, ds,$$

and since the ring is actually continuous at A we have as one necessary relation between the three unknown quantities which appear in the expression for M

$$\int_{A}^{A'} (M/EI)\, ds = 0. \qquad . \quad . \quad . \quad . \quad (2)$$

Again, if every part of the ring were rigid except the length δs between B and C, the bending in this length would move A′ relatively to A through a distance

$$\overline{BA} \;.\; \delta\psi = (M_B.\overline{BA}/EI)\; \delta s, \text{ by (1)},$$

in a direction perpendicular to BA. If axes Ax, Ay be taken along the tangent and radius at A, as shown, this displacement of A′ has components

$$\delta u = (M_B \delta s/EI)\,.\,\overline{BA}\cos B\hat{A}O = (M_B y_B/EI)\,\delta s$$

along Ax, and

$$\delta v = -\,(M_B \delta s/EI)\,.\,\overline{BA}\sin BAO = -\,(M_B x_B/EI)\,\delta s$$

* "The Stresses in a Circular Ring." Selected Engineering Paper No. 12 (1924) of the Institution of Civil Engineers.

† *Ibid.*, p. 4.

along Ay. The resultant displacement of A′, due to the bending in every part of the ring, is thus defined by the components

$$u=\int_{A}^{A'}(M.y/EI)\,ds, \qquad v=-\int_{A'}^{A}(M.x/EI)\,ds,$$

whence we have, as further conditions of continuity at A,

$$\int_{A}^{A'}(M.y/EI)\,ds=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\int_{A}^{A'}(M.x/EI)\,ds=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

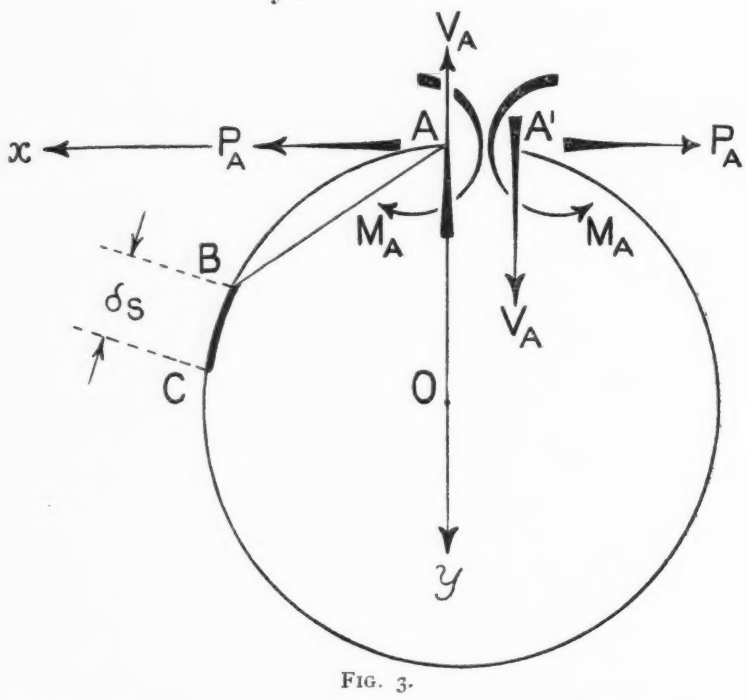

FIG. 3.

The integrals involved in (2)—(4) can be evaluated without difficulty by approximate methods, and the three linear equations thus obtained will serve to determine the unknown actions M_A, P_A and V_A; the calculation of stresses then becomes a statical problem. It will be noticed that if the quantity EI is constant throughout the ring, equations (2)—(4) take the simpler forms

$$\int Mds=\int Mxds=\int Myds=0.$$

10. *System B.*—The other two systems of construction are evidently such as will be most conveniently treated by the methods customarily employed for frameworks; that is to say, in the first place we assume the joints to be free and concentrate on the primary stress-system. The first question which arises is the order of the redundancy involved. It is known that in a plane frame-

work the number of redundant members, out of a total number m, is given by r, where

$$r = m - (2j - 3), \quad . \quad . \quad . \quad . \quad . \quad (6)$$

j being the number of the joints.* Now in System B, if n be the number of sides in either polygon, we have at once

$$\left.\begin{array}{l} m = 4n, \quad j = 2n, \\ \text{and hence} \\ \qquad r = 3. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This is so low an order of redundancy that it is evidently quite unnecessary to devise approximate methods of treatment. In fact, for load-systems due to gravity the effective redundancy is of order 2, owing to symmetry: thus, if we treat the members AB, AB′, CC′ in Fig. 2 as redundant, we may assume, when we come to investigate the effect of unit tensions in the ordinary way, that unit tensions are imposed in AB and AB′ simultaneously. The stresses in the simple framework obtained when these three members are removed can be found at once by graphical methods, and the subsequent procedure is either by "comparison of deflections," as described by W. S. Farren in R. and M. 769, or by the method of least work, as described in Appendix VII. of the Airship Stressing Panel's Report.†

11. *System C.*—Similar remarks apply to this system. If the number of sides in the outer polygon is n, we have at once

$$\left.\begin{array}{l} m = 2n, \quad j = n, \\ \text{whence, by (6),} \\ \qquad r = 3, \text{ as before.} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Here again, the order of the redundancy is reduced to 2 in the symmetrical case, and in this instance the members which will be most convenient to treat as redundant will probably be those lettered AD, AD′, B′C, BC′ in Fig. 2. All four of these members can be omitted in the symmetrical case, although the frame is thereby rendered no longer stiff, because the symmetry of the loading prevents collapse; and in the second stage of the investigation it may be assumed that unit tensions are imposed in each pair of members simultaneously. In an unsymmetrical case, three of the members must be treated as redundant.

12. Another point of difference will arise in this system if (as will probably be the case) the chord-members, being designed purely for tension, are not connected to one another at points where they cross; and unless it were desired to suspend weights from the junction points, it would appear better in every way that they should not be so connected. The possibility of members crossing without interaction on one another is not contemplated in most text-books where graphic methods for the treatment of frameworks are described. But it will readily be seen that interconnection of two crossing members can have no effect on the tensions imposed in them, so long as the framework is non-redundant; and hence, when the redundant members have been assumed to be removed, and the stresses in the remaining members are being found by means of a force-diagram, we may letter the skeleton diagram (in "Bow's notation") as though each point of crossing were a free joint connecting four distinct members: it is only in the subsequent (strain-energy) calculations that we have to remember the absence of any real connection,—and here no difficulty arises.

13. One last consideration remains to be mentioned. We have already suggested (§§6 and 12) that it should be practicable to design the chord-members in System C for tension only. Treating the system by orthodox methods, as is recommended here, it is an easy matter to determine what amount, if any, of self-straining must be imposed in order that this aim may be attained. The

* *Cf.* R. & M. 800, Appendix VII.
† R. & M. 800.

required alteration in procedure is as follows:—When we have determined the effects of unit tensions in the redundant members, we do not, as previously, make a comparison of deflections to find the actual tensions which must exist in those members in order that the structure may be initially free from strain: instead, we impose such values for these tensions as will make the resultant action tensile in every chord-member, and we use our comparison of displacements to determine the amounts by which the redundant members must be "short" initially,—*i.e.*, the amount of initial self-straining required. Equal tensions should be imposed on all the redundant members, in order that the initial stress-system due to self-straining may be symmetrical; and when this is done it will be found that the required calculations are reduced to an amount which corresponds to an order of redundancy 1.

General Considerations

14. The methods advocated in this note for dealing with the transverse frameworks of airships are evidently far wider in their scope than any generalised formulæ which could be devised by methods of the kind which the Airship Stressing Panel found necessary for dealing with the hull structure as a whole; moreover, they give the designer a far better insight into the effects of modifications in his distribution of loads. They are rendered possible by the low degree of redundancy which is exhibited by Systems B and C of Fig. 2; and it seems highly desirable that practical systems of bracing should be designed to have this low redundancy, in order that the advantages of exact calculation by simple methods may be retained. We can then deal with cross-sections of any shape whatever (it is not necessary that the polygon shall be regular), and the members can be varied in strength as may be found desirable.

APPENDIX II.

Note on the "Helical System" of Lift Wires for Rigid Airships

(*Unpublished Paper No. T.2035 (February, 1925) of the Aeronautical Research Committee.*)

Introductory

1. A fundamental problem in rigid airship design is the provision of suitable means for transmitting the lift of the gasbags to the joints of the hull structure. The longitudinals are long and slender members, liable to be subjected to heavy compressive forces arising out of the bending moments which are imposed upon the structure as a whole, and it is not desirable that they should be called upon to act in a second capacity, as girders transmitting lateral forces applied throughout their length. What seems wanted is an entirely separate system of support for the gasbags, attached to the structure, so far as may be practicable, only at its main joints; the most promising of the schemes so far suggested appears to be that known as the "helical wiring system," in which a netting of approximately rectangular mesh is constructed by running wires in two systems of intersecting helices (*i.e.*, right- and left-handed) from one transverse frame to the next, and connecting each pair of wires where they cross.

In Fig. 1, ABCD and A′B′C′D′ represent (diagrammatically) two adjoining transverse frames, and AA′, BB′, CC′, DD′ are four of the longitudinal members which connect them; it must be understood that BB′, CC′ are *not* adjacent longitudinals, the angle χ subtended by BC at the axis OO′ being of the order of 45 degrees or more. The helical wiring system is constructed by running wires from B to C′ and from B′ to C, and similarly between other joints separated by an equal angular distance χ. If these wires were taut, they would of course lie

along the surface of an hyperboloid of one sheet having OO′ as axis; by lengthening them until they lie along the surface of a circular cylinder, they can be given a curvature which enables them to withstand the radial pressure of the gasbags without developing excessive tensions, and in this way a deep instead of a flat catenary suspension can be provided for the gasbags. The importance of the deep suspension lies, of course, in the fact that it does not impose heavy compressions on the longitudinals: in fact, by choosing a suitable pitch for the helices it should be possible to arrange that such compressions as are imposed will be more or less exactly neutralised by the pressure of the gasbags upon the bulkhead netting.

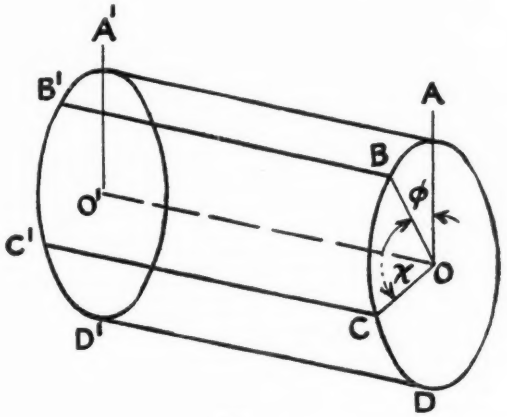

FIG. 1.

2. As an example, let us consider a netting of this kind, arranged to lie on a circular cylinder of radius a, in which the pitch of the helices (if continued completely round the cylinder) is l; and let us suppose the gasbag to be inflated with *air*, so that the radial pressure is uniform. Evidently the tension in every wire will have the same value T; and if each of the two crossing systems of helices comprises n wires, the total axial pull of the netting on the transverse frames will be $2nT\cos\theta$, and the total pull across a generator of length l in the cylindrical surface will be $2nT\sin\theta$, where $\tan\theta = 2\pi a/l$. If p be the internal pressure, we have as the equation of equilibrium

$$2nT\sin\theta = p.a.l, \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

and the net pull on the transverse frames (to be resisted by compression in the longitudinals) is

$$\begin{aligned} Z &= 2nT\cos\theta - \pi a^2 p, \\ &= 2nT\cos\theta\,[1 - (\pi a/l)\tan\theta], \text{ from (1)}, \\ &= 2nT\cos\theta\,[1 - (2\pi^2 a^2/l^2)]. \qquad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

The stress imposed upon the longitudinals is thus compressive or tensile according as l is greater or less than $\sqrt{2\pi}a$.

3. In practice, where the gas pressure is exerted by hydrogen having a free surface either within or, at most, slightly outside the cylindrical surface, the tensions in the different wires will not be equal, and the problem of determining their intensities acquires a practical importance, since the designer must provide against any compression which they may exert upon the longitudinals.

I have endeavoured recently to supply the required analysis; but it would seem that an exact solution of the problem will be very difficult to obtain, and the results, in all probability, too complicated to be practically useful. What one would desire of the netting is that it should maintain its shape (or rather, that it should not bulge beyond the cylindrical surface for which it was designed) under the pressures exerted by a gasbag ranging between the completely full and the partially empty (ground level) conditions; and of the analysis, that it should furnish reliable estimates for the wire tensions under any specified conditions. I am inclined to think that both requirements can be satisfied within the accuracy necessary for practical design; but it is difficult to frame a convincing argument to this effect without actually completing the analysis, and since I am not able to do so, I have written this note with the idea of calling attention to the problem. In my view, it is one which merits careful experimental study; for the formulæ which I have obtained by approximate methods require confirmation before they can be trusted for the purposes of design, and the extent to which they may be in error is a question of the first importance.

Statement of the Analytical Problem. The Extensibility of the Wires may be Neglected

5. We may assume the netting to be designed in such a way that one of its possible configurations is that considered in §2. In the notation of that section, if the cylindrical netting were unrolled and laid flat, it would have meshes of diamond shape, with diagonals of length l/n (parallel to the axis of the cylinder) and $2\pi a/n$ (in the circumferential direction). The absolute size of mesh (*i.e.*, the value of n) is not important, and if it assists our analysis we may simplify the problem by treating n as infinite, thus permitting the use of continuous functions and operators; but it must be noted that *we are not at liberty to replace the netting by a membrane,* since we should thereby fail to take account of the freedom with which the angles of a mesh (as distinct from the lengths of its sides) can alter as the configuration is changed.

6. If the netting were in fact replaced by a membrane (*e.g.*, of fabric), the extensibility of this membrane would at once become a factor of the first importance; for the cylindrical membrane, if imagined to be completed by discs stretched across the transverse frames at its ends, would in effect constitute a completely closed " shell," which we know (from Jellett's theorem*) to be incapable of distorting without undergoing superficial extension. On the application of internal pressures, intense stresses would be imposed along the straight generators, and the longitudinal members of the hull would experience correspondingly heavy compressive forces, of which the intensity would be dependent on the extensibility of the membrane, and so very uncertain. The great advantage which the netting possesses lies in the fact that the wire tensions will be sensibly unaffected by the extensions which result from load; without serious loss of accuracy, the wires may be treated as inextensible.

7. The analytical problem which we have to solve is the determination of the tensions in the netting when a given system of radial pressures is imposed by the contained gasbag; such pressure systems will, of course, be of hydrostatic type. Confirmation of the conclusion reached above, that it is unnecessary to take account of the extensibility of the wires, may be obtained by considering the numbers of the unknown quantities and of the equations available for their determination, just as is done in examining whether a given framework is " simple " or " redundant."

* *Trans. Roy. Irish Acad.*, Vol. XXII (1854), pp. 343-377. Some notes on this paper were given in a paper (T. 1345) " On the Shapes of Parachutes and Other Non-Rigid Envelopes," communicated in 1919. *Cf.* footnote, § 3.2.

Netting of Finite Mesh. The Problem Corresponds to that of a "Simple Framework"

8. Let the number of complete meshes in the length of a bay be m: as shown by the numbers near the bottom of the diagram, the value of m in Fig. 2 is 4. As in §2, let the number of wires in each helical system be n ($n=8$ in Fig. 2). Then, starting from the left-hand transverse frame (assumed fixed and

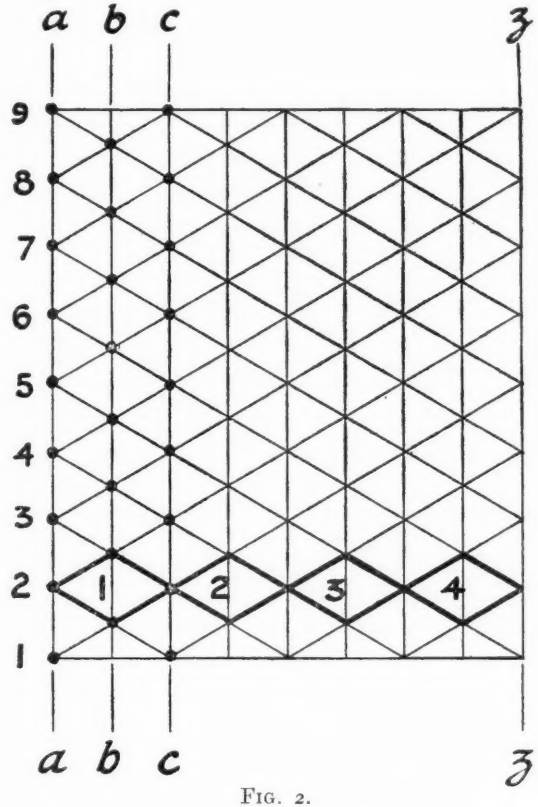

FIG. 2.
Development of Netting.

rigid) and treating the wires as inextensible, we see that one arbitrary displacement (a rotation about the line aa) can be given to each of the n nodes of the netting which initially lie in the line bb; n more arbitrary displacements serve to fix the positions of the nodes in the line cc; and so on. If the nodes which are attached to the right-hand traverse (zz) were unconstrained, we should have in all $2mn$ arbitrary displacements in a completely general displacement of the netting: the fact that the nodes on the right are in reality prevented from moving in any direction introduces $3n$ "conditions of fixity," and so reduces the number of the possible independent displacements to $(2m-3)n$.

In addition to these displacements, the unknowns comprise $4mn$ unknown tensions in the sides of the meshes, and thus the total number of the unknowns is $3(2m-1)n$. To fix their values we have three equations of equilibrium for

each of the $(2m-1)\,n$ unconstrained nodes. The problem thus resembles that of a simple framework, no considerations of elasticity being involved.

9. In the foregoing paragraph, no restriction was imposed on the magnitude of the distortion. If this be assumed infinitesimal, we may neglect the changes in tension which result from it; the tension then becomes constant throughout the length of any given wire, so that the unknown tensions are $2n$ in number; the total number of unknowns becomes $(2m-1)\,n$, and an equal number of equations of equilibrium (one for each joint) is available for their determination. The main conclusion is unchanged, and the number of simultaneous equations is still too great for a complete solution to be practicable.

Netting of Infinitesimal Mesh. Outline of a Treatment of Small Distortions

10. Letting n and m increase without limit, we arrive (by a process frequently employed in Lord Rayleigh's "Sound") at a problem which can be treated in terms of continuous functions. I have tried to develop the analysis of small displacements from the cylindrical form, defining the new configuration in terms of radial, tangential and axial displacements u, v, w (as in the elastic theory of cylindrical shells), and imposing the condition that elementary lengths in the paths of the helical wires are unextended. This restriction enables us to express u, v and w in terms of a single function; but the resulting equations are very complicated, and appear unlikely to lead to useful results: moreover, it must be remembered that no justification has yet been obtained for imposing *à priori* any limitation on the magnitude of the distortion.

An Approximate Solution for the Parallel Bay

11. A more practical line of attack can be developed as follows:—Assuming the cylindrical form to be maintained, so that the tension along any helical wire has a constant value,* we express this tension as a function of the angular position of its middle point. When the function is specified, the condition for radial equilibrium of a given mesh defines the radial pressure which must act on that mesh in order that the cylindrical form may be exactly maintained. We choose the function so that this required pressure distribution conforms as closely as possible with the distribution of the pressure actually applied, and we assume that the pressure which remains is in fact balanced by some *slight* distortion from the circular form.

12. As an example, take the case of a full gasbag in a horizontal bay. Measuring angular position from the vertical plane AOO′A′ of Fig. 1, we have for the imposed radial pressure at an angular position ϕ,

$$p_\phi = \alpha + \beta \cos \phi, \qquad . \quad . \quad . \quad . \quad . \quad (3)$$

where α and β are constants.

Now consider the two wires which cross at a node in the netting distant x from the centre of the bay and having an angular position ϕ. The middle points of the two wires have angular positions $(\phi + x/a \tan \theta)$ and $(\phi - x/a \tan \theta)$; hence, if the tension in any wire is defined, as in §11, by the expression

$$T = F(\phi), \qquad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

then the tensions in the two wires which cross at (x, ϕ) are respectively

$$\left.\begin{aligned} &T_1 = F\{\phi + (x/a) \tan \theta\} \\ \text{and} \quad &T_2 = F\{\phi - (x/a) \tan \theta\}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

* If two wires cross, and neither wire changes its direction at the junction, it is easily seen that neither tension has its value altered there.

Now the equation of radial equilibrium is given approximately, when the mesh of the netting is small, by

$$(T_1+T_2)\sin\theta=aLp, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where L is the axial distance between adjacent wires of the same helical system, and p the radial pressure required to maintain equilibrium. At the centre of the bay, where $x=0$, we have from (5),

$$T_1+T_2=2F(\phi): \quad . \quad . \quad . \quad . \quad . \quad (7)$$

hence, if the applied pressure is exactly balanced at the centre of the bay, we have, from (3), (4) and (6),

$$T=F(\phi)=(aL/2\sin\theta)(\alpha+\beta\cos\phi). \quad . \quad . \quad . \quad . \quad (8)$$

It follows from (5) and (6) that the pressure required for equilibrium at the point (x, ϕ) is given by

$$\begin{aligned} p&=\tfrac{1}{2}\left[\{\alpha+\beta\cos(\phi+x\tan\theta/a)\}\right.\\ &\qquad\left.+\{\alpha+\beta\cos(\phi-x\tan\theta/a)\}\right],\\ &=\alpha+\beta\cos\phi\cos(x\tan\theta/a). \quad . \quad . \quad . \quad . \quad (9)\end{aligned}$$

It is less than the actual applied pressure p_ϕ by an amount

$$p'=\beta\cos\phi\{1-\cos(x\tan\theta/a)\}. \quad . \quad . \quad . \quad . \quad (10)$$

13. As stated in §11, we may assume that the error represented by this pressure p' is in fact neutralised by a slight distortion of the netting from the cylindrical form. By a suitable alteration in the formula (8), from which our estimates for the wire tensions are derived, we can obtain an error of opposite sign, and it seems fairly safe to assume that the actual wire tensions will lie between limits obtained in this way.

Thus, if we replace (8) by the formula

$$T=F(\phi)=(aL/2\sin\theta)(\alpha+\gamma\cos\phi), \quad . \quad . \quad . \quad (11)$$

where γ is a constant not yet determined, the pressure required for equilibrium is given by

$$p=\alpha+\gamma\cos\phi\cos(x\tan\theta/a), \quad . \quad . \quad . \quad . \quad (12)$$

and the error is given by

$$p'=\cos\phi\{\beta-\gamma\cos(x\tan\theta/a)\}. \quad . \quad . \quad . \quad (13)$$

Then, whereas the quantity in twisted brackets in (10) is necessarily *positive*, the corresponding quantity in (13) will be necessarily *negative* if γ is fixed by the relation

$$\beta=\gamma\cos(b\tan\theta/2a), \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where b is the length of the bay; so it would seem that the wire tensions may be estimated from (11), where γ has a value intermediate between

$$\beta \text{ and } \beta\sec(b\tan\theta/2a). \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The Case for Experimental Investigation

14. The methods of the preceding paragraphs could probably be extended without difficulty to tapered (*i.e.*, conical) bays. But they involve several assumptions of doubtful validity, and it would seem that the first requirement is an experimental test of the approximate formulæ suggested, by means of a water model: in this way many questions could be investigated (such as the effect of partially empty gasbags, or of tilting the ship) which would involve very great analytical difficulties. This note is intended merely to present a case for such experiments: they would not be difficult to carry out, and in my opinion would provide information likely to be of great value, if the helical system of wiring is to be adopted in future designs.

THE RELATIONSHIP OF PHYSICS TO AERONAUTICAL RESEARCH

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DIRECTOR OF SCIENTIFIC RESEARCH, AIR MINISTRY.

Aeronautics is a youthful study—a mere Cinderella of the Sciences, distinguished from her sisters by her tender years, by her guileless and impetuous disregard of the pinch of the financial shoe, and by her beauty.

That it is Applied Physics is to me the most inspiring definition of engineering; and if this be true for engineering in general, as I think it is, especially true is it of aeronautics. To illustrate the close association of these new studies with the work of physicists, I shall select a few of the more striking results of recent research.

Aviation is now entering upon a new and intensely interesting phase—one which will call for every scientific resource at our command. The materials of construction are changing: wood is giving place to metal. The engine proves to have most unexpected possibilities ahead of it through the increase of intake pressure; whilst the very lifting structure itself promises to change, for some purposes at least, from linear motion to rotary.

In aeronautical work there can be no doubt that science has been very considerably stimulated by problems which have arisen directly from aeronautical engineering. The investigations of those physicists who have in recent years done such brilliant work on the mathematical theory of fluid motion has been stimulated in no small degree by the results obtained either in the wind channels of our aerodynamic laboratories or in free flight on the full scale. Work of this nature has been rendered the more possible by reason of the response made by the Universities of the British Empire to the call for scientific men to enter aviation during the war of 1914-18; and, as in private duty bound, I would refer especially to those Cambridge men who by their courage, scientific insight, and daring have left so great a mark on aeronautical studies. Fortunately for aviation the services of these men are, in many instances, still devoted to these great ends.

Aeronautics by its youth escaped the medievalism of the old time "practical engineer;" it was launched well in the modern scientific path by the formation some 17 years ago of the Advisory Committee for Aeronautics, which after two phases of re-adjustment is still with us as the Aeronautical Research Committee. When I first knew it, it was presided over by that kindly and gracious being the late Lord Rayleigh; in those days the field of work was small enough for all matters to be dealt with by the single Committee. Now the scale is so much greater that many sub-committees and panels have to share its responsibilities. The change is so great that I sometimes feel inclined to quote Miss Beale, the famous headmistress: "I was born in the middle ages and I have lived through the Renaissance." I quote from memory, but I think I have it right.

The latest phase of that Renaissance is the organisation under the Air Ministry of a separate scientific research department to ensure that scientific investigations shall receive their due and be vigilantly guarded against any competition for money, staff, or facilities, by the seemingly urgent claims coming

from other quarters. In some measure this is following the Admiralty lead of a few years back when a former distinguished fellow student of mine, Mr. F. E. Smith, was appointed head of a small staff to supervise scientific research work for the Admiralty.

When a Government wishes to create a scientific research staff for investigations in aeronautics, or in any other subject, there are special difficulties to encounter. The greatest of these is the break with tradition. Governments are used to dealing with large staffs and have evolved special methods, doubtless useful in considerable measure, for dealing with them. But a scientific staff is a new sort of staff and a scientific research staff still newer. For Government Departments to adopt University methods *in toto* would scarcely be practicable, but it is nevertheless necessary to go a good way in that direction, especially in regard to freedom of work and the avoidance of the burdensome regulations usually associated with government enterprises. I am glad to say that the event has shown that satisfactory arrangements in this respect can be made. To an increasing degree it has been possible to facilitate the interchange of research workers between the Universities and the Government research laboratories, and to ensure satisfactory arrangements for carrying out research work in the latter establishments. There is now something approaching uniformity of conditions throughout large staffs such as those at the National Physical Laboratory and the scientific research staffs under the Admiralty and Air Ministry, and facilities are available for the promotion of workers from the one staff to the other.

The scientific research staff of the Air Ministry closely resembles the corresponding staff at the National Physical Laboratory in its method of recruiting, its scales of pay, and its conditions of work. This staff is about eighty in number, and of this general "pool" some four-fifths work in the Laboratories at Farnborough, and the rest at such stations as Martlesham, where the performance of new airplanes is studied, Felixstowe, where corresponding work is done for seaplanes, and the Air Ministry Laboratory in the Imperial College of Science, where special problems, mainly physical and chemical, are studied. It is normally required that entrants to this staff shall have received a first or second class honours degree in science or mathematics and have shown aptitude for research work.

Every endeavour is made to give free scope to the researches of those workers who have shown over a series of years that they have a special flair for fundamental investigations. That this can be done even under the so often maligned Government organisation is demonstrated by the work of Glauert, of Griffith, and of those others whose names, by reason of the nature of their work, are less well known in the world of physics. It often strikes me as a great misfortune that the confidential nature of some investigations must, in the nature of the case, prevent a mass of most interesting scientific work from coming before the scientific world. Some of this work may no doubt be released at a later date, but much of its interest will by then have evaporated by reason of the cognate researches of outside workers. A good deal of research work on instruments comes under this heading, and such work as that on the ballistic qualities of falling bombs, carried out many years ago in a half-mile deep coal mine at Rossington, the corresponding trajectories under water and the propagation of explosive waves through the sea studied in connection with anti-submarine work. This concerns only the smaller section of the research work however; the greater part is freely published through the medium of the reports and memoranda of the Aeronautical Research Committee and in other ways.

The examination of the flow around aerofoils and of the fluid forces brought into play has naturally formed one of the chief items of physical research in the wind channels of the aerodynamic laboratories throughout the world. The

simpler measurements usually made on aerofoils are the lift and drag coefficients and the motion of the centre of pressure. These, the ordinary methods of classical hydrodynamics are not capable of predicting from theoretical considerations alone, since the mathematics deals only with the motion of a fluid in which viscosity plays no part.

In the case of a simple mathematical shape such as a cylinder placed at right angles to the flow, the positions of the streamlines, as determined for an inviscid fluid, differ appreciably from those which would arise in a viscous medium, such as air. In the former fluid moreover no resultant force acts upon the cylinder, whereas measurements made in an air channel show that considerable forces are brought into play. Indeed without such forces flying would not be possible.

If now to the streaming of the inviscid fluid past the cylinder there be arbitrarily added a " circulation " of the fluid around the cylinder, there at once comes into existence a resultant force at right angles to the stream. This has an immediate analogy with the lift force experienced by an aerofoil, and with the propulsive force produced by the action of the wind on the rotating cylinder of the Flettner rotor ship.

It is seen, therefore, that by the addition of the idea of a " circulation " around the body, a possible explanation is given for the existence of lift forces, and in recent years, by the employment of this convention, successful efforts have been made actually to predict the lift of certain forms of wing section. Such predictions can it is true be criticised on the ground that they are based on an illogical, and at first sight, somewhat far-fetched summation of a theory based upon the properties of an inviscid fluid with a purely hypothetical convention as to what it is, or may be, happening at the boundary layer between the fluid and the body. An inviscid fluid would at this boundary slip past the surface of the model without retardation and the slipping is equally there even when the idea of a " circulation " is added; in practice, however, it does not appear that slip of such a character has any real existence since there appears to be an exceedingly thin layer of the fluid in which one side, so to speak, is in steady contact with the model and the other with the rest of the fluid medium—a sort of self-lubrication.

The theory itself was built upon the work of many investigators, and among that many I would like specially to refer to the work of Lanchester, Joukowsky and Prandtl. As a result therefore of the work of these and other pioneers, we find it useful in aerofoil design to study the effect of imposing upon the motion of an inviscid fluid a " circulation " of unknown origin around the aerofoil of sufficient amount to carry what is known as the " stagnation point " to the trailing edge of the wing. " Circulation " must have a physical existence since velocity is greater above the wing than it is below; though this real circulation is a circulation with no slip, whereas the mathematical circulation has slip. Hence the rather amusing situation arises of adding to the mathematical study of streamlines a conventional motion which could not really arise in an inviscid fluid! So long, however, as the limitations to the theory are borne in mind no harm can be done, whilst the results of such mathematical work are of the greatest utility as a first approximation as to what is happening. It has indeed been suggested by Southwell that the whole of the lifting power of an aeroplane wing can be appropriately referred to what goes on in this " skin " of perhaps only a few thousandths of an inch in thickness. The general effect was pointed out by Bairstow (Royal Aeronautical Society, 1925), that in the circulation theory of aerofoils developed by this school, the conception of the aerofoil itself is in effect modified so as to include the " skin." I shall refer further to the consequences which follow from this convention and its mathematical treatment in a moment.

It is natural to ask, when endeavouring to design an aerofoil of improved efficiency, whether there is any theoretical limit beyond which improvement would be physically impossible. In the study of the internal combustion engine, it has been proved that no engine can have a better thermodynamic performance than that given by what is known as the " air standard." If some such standard could be discovered for aerofoils it would be no small convenience. In order to see how far such could be the case, consider that what happens to the air in the neighbourhood of the wing of an aircraft in flight; the circulation around the wing section becomes at the wing tips two trailing vortices, giving, if the parallel be permitted, the effect of the heavy moustaches of a dragoon! If the wing were of infinite span the effect of the trailing vortices would be negligible in comparison with the forces acting on the wing as a whole, and the drag force would then be limited to what is known as the " profile drag." The consequence of the limited span of an aerofoil of finite aspect ratio is to add to the profile drag, an " induced drag; " and the total drag of the wing is made up of the sum of these two. There appears to be good reason for believing that in the neighbourhood of minimum drag the normal components of the pressure around the aerofoil account for a considerable proportion of the profile drag, the remainder being contributed by the surface tractions. Work at the National Physical Laboratory has shown that the average intensity per unit area of the drag components of such surface tractions is of the same order of magnitude as the surface friction on a long flat plate. For a wing therefore of definite span and chord it would be quite simple to assess the amount of the drag due to such surface tractions, and this necessarily represents an inferior limit, but so inferior as to be, I fear, but relatively little guide as a standard of efficiency. There is room here for a further study of these matters by physicists and mathematicians.

One great merit of Prandtl's work is that his theory enables the lift of a wing of limited span to be deduced from the known performance of a wing of the same cross-section, but of unlimited span; in other words, that the results of three dimensional flow can be deduced from the known two dimensional. The two dimensional flow can be studied in two ways; experimentally, by means of a model wing section stretching right across the wind channel; and mathematically, by the method initiated by Joukowsky, who found a way of predicting these forces by the mathematical step of converting a circle into the desired aerofoil shape by means of a suitable conformal transformation. For a limited number of aerofoil shapes it is therefore possible to deduce theoretically the performance for two-dimensional flow, whilst by the Prandtl theory it is possible to convert these results from two dimensional to three dimensional conditions.

These theories have an application also to airscrews; experimental work has shown that the theoretical conclusion from the vortex theory that the lift and drag of each element of an airscrew blade may be calculated as if it were part of an aerofoil of infinite aspect ratio has been verified. Attempts have also been made in the wind channels in many countries to study the performances of an airscrew when acting as a windmill.

This brings me naturally to the consideration of the rotating wing as distinct from the fixed wing hitherto used in aircraft to which we have become accustomed. Aeroplanes of the present day type are capable of a steady motion known as a spin; in this they follow a kind of corkscrew path in which the wings have a rotating as well as an advancing motion. I am unable to feel certain that there has been any occurrence in this country of an entirely " flat spin," although the phenomenon has been reported from the United States and elsewhere. In this case, the machine while on a level keel rotates about a vertical axis, slowly descending as it does so. The descent is so slow that there is little likelihood of injury to the occupants even if the evolution be persisted in until contact with the ground. It may in fact be the case that with fixed wing machines, this evolution is worthy of serious study as a means of landing in an emergency!

Many attempts have been made to build aircraft, known as helicopters, fitted with rotating wings driven by one or more engines. Little success has, however, been met with in spite of the very considerable expenditure. But in the last year or two, real success has been achieved by Senor de la Cierva with his "autogiro;" this machine is not strictly a helicopter, however, since the windmill is not power driven. But I think there is little doubt that it is through the autogiro or, in English, the gyroplane, that we have the best means of studying the performance of all rotating wing machines.

One of the difficulties with a rotating wing is that the wing which is moving in the direction of flight has a higher airspeed than the retiring wing, and this leads to much greater lift on one side of the machine than the other. Mechanisms to balance these forces mechanically are most complicated to build and still more complicated to control. Cierva cut the knot of this difficulty by hinging his blades close up to the vertical shaft. This meant that the advancing blade rose, thereby reducing the relative incidence, whilst the retiring blade fell, thereby increasing the relative incidence; thus the wings oscillated in a vertical plane during each rotation and presented some analogy to the flapping flight of birds. Centrifugal force prevents the wings from folding up about their hinges.

Cierva designed his machine on the basis of wind channel work carried out at Quatro Vientos at Madrid. Those experiments and the subsequent full-scale work showed two remarkable characteristics. First that the windmill rotated in the opposite direction to that which might otherwise have been expected; and secondly, that the vertical speed of descent when the engine was stopped was much less than had previously been expected from such wind channel work as had been done. The four blades of the gyroplane are set at a pitch angle of opposite sign to that of the ordinary windmill still to be seen in our agricultural districts. It is puzzling at first to see how this can be the case, but the solution is best attained by considering what is happening to the gyroplane when descending vertically in still air. The motion of the air relative to the blades is then made up of the peripheral speed of the blades combined with the vertical speed of descent, giving a sloping component the tangent of whose angle to the horizontal is equal to the vertical speed divided by the peripheral speed. Now the lift on the blade will be at right angles to the relative wind and the drag will be along the wind. The resultant of lift and drag is a force of which the direction slopes behind the lift line, and if the blades are mounted on a vertical axis this resultant may be tilted ahead relative to that axis, or tilted backwards, depending upon such factors as the lift-drag ratio for that particular wing section at that particular angle of incidence. If this resultant force is tilted forward there will be component force hastening the rotation of the windmill, whereas if it is tilted backwards there will be a retarding force. Equilibrium is attained when the resultant force becomes parallel to the axis. What happens in practice is that there is a critical speed of rotation which when exceeded builds up by itself automatically, but which if not attained leads to the cessation of rotation when the driving effort is relaxed, hence if the wings are brought up to speed they may accelerate or decelerate depending on what the speed is, but once they pass a certain critical angular velocity they speed up to a steady state in which they are in a kind of locked condition, which fortunately greatly increases the safety of flight of gyroplanes. When an endeavour is made to compute in this fashion the velocity of descent for a given loading, the main difficulty is to determine the speed of air through the disc. Attempts have been made to do this theoretically by Glauert and Low, and we hope that the problem will be solved before long. Owing to wall interference in the wind channel tests that have previously been made on such windmills, or for some other reason, their capacity for slow descent under load was underestimated. This under-estimate was first realised as a result of the tests carried

out at Farnborough last summer with a Cierva gyroplane in which the vertical velocity was some 15 feet per second only, about half the speed of fall of a parachute carrying the same load and having a diameter equal to that of the windmill.

Full-scale tests on aircraft and aircraft components usually require a greater space than can be given in a laboratory, and in consequence the physical investigations arising therefrom need to be carried out through the medium of models. It is no difficult matter to make an accurate scale model of any form of aircraft and to provide a wind channel of such dimensions as will make the test reasonably representative. That is relatively simple, but a difficulty soon arises through what is known as "scale effect." The study of scale effect is intricate and much work has yet to be done. But it has been proved that model tests in the wind channel will represent what happens on the full scale provided that a certain physical ratio is preserved; this ratio is the product of length by velocity, divided by the kinematic coefficient of viscosity. This ratio is a non-dimensional one and it is coming to be known as the "Reynolds number," in memory of that famous physicist Osborne Reynolds. Model tests in wind channels, as we at present know them, give a value of the Reynolds number of but one tenth of that appropriate to full scale. This is unfortunate, since it means that within the range of velocities available in the ordinary wind channel only a very small portion of the correction for the change of the characteristic under measurement with the Reynolds' number can be studied; hence, to deduce full-scale results from wind channel experiments may mean extrapolation in the ratio of something like 10:1. In a moment I shall refer to a later form of wind channel which aims at avoiding this difficulty. In the meantime it is due that reference should be made to the work of Prandtl in enabling us to correct the wind channel measurements for wall interference as well as for aspect ratio.

A limit to the air velocity in the channel is imposed by the considerable increase of horse-power required for such operation; there is also an obvious limit to the dimensions of channels, and it follows that the only opportunity of obtaining a Reynolds' number comparable with that applicable to full-scale conditions is by means of an increase in density of the air, or else of its replacement by some fluid having a lower kinematic viscosity. Much interest has been taken in the pressure channel recently built at Washington which is capable of being operated at an internal pressure of 20 atmospheres, so realising a Reynolds' number equal to that of full-scale work, when allowance is made for the fact that a pressure tunnel is best run at about half the airspeed of an atmospheric tunnel so that the amount of the forces acting upon the model may be kept within bounds. It is too early to say whether this channel has entirely fulfilled its constructors' hopes, but from such tests as have been reported it seems that the method of testing rendered possible by its use will prove to be of very great utility. It is true that even if no pressure channel is available the experimental work at a high Reynolds' number can be carried out by full-scale flying; this, however, is an exceedingly costly alternative and once the need for work at high Reynolds' numbers is established it will be very much cheaper to build a pressure channel.

Another form of model testing is that carried out in the Yarrow tank at the National Physical Laboratory. This tank was originally built for the testing of ship models, but in recent years it has also been used for work on models of seaplanes. Investigations of this sort became specially necessary when the sea-worthy characteristics of seaplanes needed study. These craft have to be designed in relationship not only to their performance as flying machines, but as to their habits when travelling on the surface of the water before they become entirely airborne. For the aerodynamic characteristics the wind channel suffices, but for the study of the period prior to flight the tank is necessary. When considering the size and speed of a tank for this purpose one is faced with the

consideration that the model must run at what naval architects term "corresponding speed" to that of the full-scale machine; speeds are said to be corresponding when the ratio of the velocity to the square root of the length of the model is equal to the velocity of the ship divided by the square root of its length. This ratio, it will be seen, is quite different from the Reynolds' number above mentioned and it leads to totally different design requirements.

It might be thought from what has been said that the main purpose of the tank is to make accurate quantitative measurements. This, however, is not the case, since probably the chief use of such test is a qualitative examination of the wave motion created by the hull, particularly in relationship to the position in which the engines and propellers will be placed.

Such are the chief instances of physical measurements with models that arise in aeronautical work, though in the allied field of the propagation of detonation waves, through either air or water, use can always be made of the remarkable pioneer work of Bertram Hopkinson. It will be remembered that he showed, during the war, that the damage done to structures by explosive waves, whether in the air or under the water, could be studied on the model scale provided that the structure were reproduced strictly to scale and that the explosive and its container were also to the same drawing and to that same scale. This remarkable discovery seems not to be as well known as it deserves to be.

The physical problems presented by the aero engine are for the most part not peculiar to aeronautics, but are part of the general study of the internal combustion engine. The use of this type of prime mover for aircraft does however present special problems owing to the urgent demand for "reliability" on the one hand and lightness on the other. Moreover, there is always the tenuity of the air at altitude to be reckoned with.

As a matter of fact the aero engine has improved enormously in recent years—not so much as the result of any scientific study as by sheer hard efficient work on the part of the engineering staffs of the engine builders.

Need for lightness of construction brings in quite other considerations. For high output, high efficiency is necessary and this calls for increased compression pressures and a consequent liability to the troubles induced by detonation. The study of detonation and the means of avoiding it are fitting studies for physicists. Equally fitting are the investigations necessary to ascertain whether the output of the engine in relation to its weight can be increased by what is known as supercharging, and if so how far in that direction it is expedient to go. All the while it has to be borne in mind that the engine must not only be capable of operating in a normal atmosphere such as that in which most internal combustion engines work, but in conditions in which the pressure may be only one-third of that at sea level and in which the atmospheric temperature may be no less than 50 degrees Centigrade below zero.

One of the most striking scientific puzzles of recent years is the effect produced on the phenomenon of detonation in an internal combustion engine by the addition to the fuel of small amounts of various substances. But to say whether the puzzle is one which comes within the realm of physics or that of chemistry is no easy matter. The answer must, I think, be that at present we do not know. What we do know is that many of the engines used in aircraft to-day in this country would exhibit the troublesome phenomenon of detonation if they were supplied with ordinary aviation spirit without admixture. To ensure smooth running the usual custom is to add to this fuel some twenty per cent. of benzol. Tests in a variable compression Ricardo engine show that by this addition the compression ratio may be raised from 5.0 to 5.8 without the occurrence of detonation. But to employ benzol in this way is not really satisfactory. Supplies cannot be counted upon in all emergencies, and in any event the freezing point is not so low as it should be for a fuel which shall be satisfactory for high

altitude work. Midgley working in the United States suggested the use of tetra ethyl lead, a highly poisonous substance reported to have been considered during the war of 1914-18 for "chemical warfare" employment, but having the remarkable property that only the tiniest proportion need be used—no more in fact than three cubic centimetres for each gallon of fuel. An engine which is run on this "doped" fuel shows at first no ill effect, but after a number of hours' running a deposit of some lead salt, probably litharge, attains sufficient bulk to prevent further running. The American workers met this difficulty by adding to the three cubic centimetres of tetra ethyl lead some two cubic centimetres of ethylene di-bromide; the intention was doubtless that the lead should be evacuated in the exhaust of lead bromide. Repeated tests have shown that the addition of ethylene di-bromide does in actual fact have the desired result. A "Lion" aero engine has been run on the test bed for 100 hours on fuel "doped" in this fashion without ill effect or loss of horse-power, though by that time the engine was dirty enough to detonate at once when fed with "undoped" fuel. Similar tests on an air-cooled engine have led to similar results.

The facts are clear enough, but what is the scientific reason for them? Here is a problem on which the assistance of physicists might well be sought. There are, of course, certain other substances which are now known to have a generally similar effect, iron carbonyl for instance; in fact a technical literature on this fascinating problem is already in existence. But the ultimate causes are unknown.

Problems more certainly physical arise from the aero engine in relation to its performance in flight as affected by the lowered pressure and temperature of the atmosphere at altitude. Since for a constant mixture strength the weight of fuel consumed depends on the weight of air taken into the cylinder, and since the cylinder is of fixed dimensions it might be thought that the power would be directly proportional to the density of the atmosphere at the point at which the charge was taken in. This, however, is not so. The air on entry is heated by the hot inlet passages and a less weight of air is taken in than would otherwise be expected—in fact the temperature at the end of the suction stroke is not far from constant. Other factors are also at work, and the combined effect of them all appears to be that on the average the brake horse-power is more nearly proportional to the atmospheric pressure than the atmospheric density.

This sensitivity of the engine to atmospheric pressure has led naturally to attempts to create an artificial atmosphere of increased density in the engine intake. A scheme of this sort was indeed mooted by Sir Dugald Clerk more than twenty years ago and was called by him "super-compression." It is now known as "supercharging" when the effort is to maintain an intake pressure at all altitudes equal to that at ground level, or "boosting" when the effort is to increase the intake pressure by a constant fraction at all heights. These developments present an infinitude of problems most of which are now beginning to be seriously tackled—their close relationship to detonation is a complicating phenomenon. There are such great possibilities in this direction that a material decrease in weight per horse-power at altitude may confidently be looked for in the not distant future.

For the world of physicists in general, aeronautics teems with problems of great complexity but intense interest which await their study.

We live in a wonderful age. Just as in the thirteenth century the splendour of life must have seemed most to surround the work of the architect, or in the fifteenth century that of the painter, so it appears to me in the present age does it crown the labour and achievement of the physicist.

STUDENT'S PAPER

EXPERIMENTAL FLYING FROM THE PILOT'S POINT OF VIEW

Paper read on March 11th, at 6.0 p.m., in the Library, 7, Albemarle Street, W.1.

BY FLYING OFFICER R. L. RAGG, R.A.F.

The term "Experimental Flying" is used to cover so many different subjects dealing with the development of practical aviation that I feel the title of this paper would be suitable for a volume of no small size, if I were to deal in detail with all it infers.

It is my intention, therefore, to restrict myself almost entirely to the practical side of aerodynamic research work, and the testing of new aeroplanes and aeroplanes fitted with experimental engines. The development and testing of air instruments, wireless sets, photography and air survey, oxygen apparatus and heated clothing for high altitude work, are all essentially the duties of the experimental pilot, and call for very accurate, careful and skilful flying; but—after all said and done—these are all accessories to flying.

I must warn you that I am not a technical man; and during the course of this paper you will find no equations whatever. I am not going to attempt to explain to you how the various observations taken on experimental flights are worked out to produce useful data. I am merely going to try to describe to you what goes on in the air during these flights, and how the observations are taken, with a short and simple description of some of the devices and instruments used in taking them.

Having told you how alpha, gamma and rho are measured in the air, I will leave you to convert them into K_L and K_D. That is a problem which I am quite incapable of solving.

Before getting on to the subject of experimental flying I should like to make a few remarks with regard to flying in general, and pilots in particular.

That strange idea, so prevalent until quite recent years, that the air pilot is a man completely devoid of nerves—a dare-devil, afraid of nothing—is, happily, now recognised as being a fallacy. He is, in reality, a very ordinary man. It is essential, of course, that he is physically and mentally fit, and well provided with that inestimable gift, commonsense. He must possess a pretty sense of judgment, especially with regard to speed and distance. His reflex action must be quick, and his senses of balance, position and "touch" should be highly developed.

In experimental flying particularly it should be remembered that steadiness and reliability are far more important than showiness and ostentation. It is not only required that an experimental pilot should be able to fly well. He should be able to take a really intelligent interest in the experiment he is carrying out, and should be capable of analysing any unusual behaviour in the air of the machine or device he is testing. He must, of course, be able to adapt his flying to all types of aircraft in all sorts of strange circumstances, as he will not infrequently be called upon to fly under the greatest of difficulties.

The adage, "Discretion is the better part of valour," was never more truly applicable than to the art of flying. How many pilots have met with disaster

through lack of discretion. It never pays to take unnecessary risks in the air—the consequences are liable to be too severe to make it worth while.

The Testing of New Types

Let us first consider the maiden flight of the first machine of an entirely new type.

Although the finished article has been designed and constructed with the greatest of care and attention to the minutest details, it is always a matter of speculation whether the first flight will be successful, and whether the machine will fulfil in the air all that is expected of it. It is an anxious time for all concerned, particularly for the pilot whose duty it is to put the machine through its tests.

We will take it for granted that the machine has been passed by the inspectors, that the engine has been run up and found to be correct, and that the pilot has acquainted himself with any peculiar details the machine may possess with regard to its petrol system, or position of controls working the tail incidence, variable camber gear, or any other unusual feature, and will assume that all is ready for its first air test.

Incidentally, it is astonishing how much the look of an aeroplane will convey to the experienced pilot. He seems to develop an " eye " for aircraft.

On a test of this sort it is wisest to begin at the beginning and leave nothing to chance. First of all the machine is " taxied " gently round the aerodrome, in order for the pilot to get a general idea of what the machine is like in motion, and to see whether the rudder is effective. (If it turns the machine moving at slow speed on the ground, it will turn it in the air.) Then the machine is taxied straight, and at fairly high speed—this is to find out how effective the elevators are, and whether the machine is unduly nose or tail heavy. The ailerons can also be tried gently while taxying fast. The sooner the tail lifts, the happier the pilot feels, for it means that the machine is nearing its flying position and will soon be off the ground. At least it proves that it is not tail heavy, and tail-heaviness is a most uncomfortable fault in a machine.

Perhaps this take-off is the most anxious time of all for the pilot, because although all the controls are effective theoretically, it still occurs occasionally that theory and practice do not agree, and for some unforeseen reason the machine takes control of the pilot—needless to say, a very trying and sometimes very serious position to be in. That is why it is always safer to work up by slow degrees, and test the effectiveness of the controls in this manner while close to the ground. It is not much use waking up in the hospital and saying, " Now *why* didn't I find out about those controls nearer the ground? " If an accident is inevitable, it is obviously less disastrous to crash at low speed near the ground than at a greater velocity from a height.

Then the engine can be opened out full, or nearly full, and the machine allowed to reach its flying speed and " hop," or fly straight for a short distance, with its wheels just off the ground.

If it passes all these tests satisfactorily, and the controls are found to function properly, it is not unreasonable to suppose that the machine is " flyable " with a certain degree of safety, and the time has arrived for it to be taken up for its first real flight.

After taxying the machine out on to the aerodrome, the pilot opens the throttle as the machine starts to roll forward gathering speed. The first thing he looks—or rather " feels "—for, is the tendency of the machine to get its tail up. A little forward pressure on the control column will help this, at the same time ensuring plenty of speed—and it should be remembered that speed, in existing aeroplanes, means control, and control stands for safety near the ground.

The machine is then allowed to leave the ground—the speed at which it takes off being noted—and is climbed until it has reached a safe altitude.

The controls can now be tried more fully, and a few gentle turns made to left and right. The machine should answer readily to the controls and should not be heavy or sluggish. It is most desirable that all the controls are well harmonised—in other words, no one of three, elevator, aileron and rudder, is appreciably heavier or lighter than the other two.

Having satisfied himself that the aeroplane is controllable in normal flight, the next thing to find is the stalling speed, with engine on and off. The engine is therefore opened out full and the control column eased back further and further, until the nose is tilted well above the horizon. Just before the stall the engine begins to labour, the whole machine seems to shudder, and a general "sogginess" or limpness is felt on all the controls. The speed at which the machine stalls with engine on is noted, and the same experiment tried with engine throttled down. The speed in the latter case will usually be a little faster than in the former, as with the engine on there is a tendency for the machine to "hang" on its airscrew. (In other words, it tries to become a helicopter.) The elevator and rudder controls will also be more effective with the engine on, owing to the slipstream of the airscrew. Some high-powered single-seaters will nearly reach a vertical position before stalling with the engine on.

It is next to be discovered whether the machine is, or is not, inherently stable on all its controls. It is therefore trimmed to fly level by means of the tail incidence adjustment. The control column is given a sharp pull backwards and released. The machine will lift its nose and take up a climbing attitude. Before the stall is reached, however, the nose will gently drop and the machine will start diving until the speed becomes great enough to lift the nose again. This "switchbacking" should slowly damp itself out until the machine eventually regains an even keel, and it can then be classed stable longitudinally. Should it refuse to return to an even keel but stalls, or persists in diving, or if the switchbacking does not damp itself out, the machine is unstable longitudinally.

The same experiment is then tried for lateral stability, and the control column instead of being pulled back is forced over to one side and released. If the machine side-slips steeply, or drops its nose and falls into a dive, it is unstable laterally. But if it oscillates from one side to the other, the oscillation becoming less until they are damped right out, it is stable laterally.

For directional stability, the machine is flown straight and level as before, but the control column is held central while the rudder bar is kicked on to one side and the feet taken off. The machine will flat-turn, but should not continue turning, or drop a wing violently and dive, but should after a moment or two resume straight flight.

Most machines nowadays are stable longitudinally and laterally, but very few can be flown with feet off the rudder-bar—the majority have a turning tendency owing to the torque of the airscrew, which varies, of course, at different speeds.

It not infrequently occurs that when the control column is left free, the machine takes up a gentle "hunting" motion (pitching fore and aft) or a slight rolling sideways. Providing oscillations are not violent, however, the machine can hardly be classed as unstable.

The controls will also be tried on the glide, with the engine throttled back. It will generally be found that the ailerons in this case will appear to be more effective—this being the result of the rudder and elevators losing the added effect supplied by the slipstream of the airscrew.

Probably during a later flight, readings will be taken of such details as the maximum speed at different heights, the most efficient climbing speed, the cruising speed, the rate of climb, and the "ceiling"—or the maximum height to which

the machine will climb. These tests will be carried out with the maximum load the machine is designed to carry. The load will take the form of sandbags, not passengers. This is in order to avoid unnecessary grave-digging in the event of any unfortunate occurrence.

Eventually " type trials " will be carried out to prove the suitability of the machine for the particular type of work for which it is intended—passenger-carrying, bombing, reconnaissance, fighting, etc.

Aerodynamic Research

Two of the greatest problems confronting the aircraft designer to-day with regard to aerodynamics are :—

(1) Scale effect, or the difference between the characteristics of the full scale machine and those of the model, and

(2) Control at low speeds.

(1) *Scale Effect.*—It has been found that although the model of a given aeroplane may be made exactly to scale in every detail and the wind channel readings taken accurately from that model, the characteristics do not agree in all respects with those of the full scale machine in flight. No definite law has yet been established whereby the performance of a full scale aeroplane can be calculated accurately from that of the model. Wind channel models are apt to be misleading at times. Take, for example, the case of thick wing sections. It has been found by experiment that the coefficients of lift and drag on a thin sectioned wing (such as R.A.F.15) are accurately measurable in the wind channel. In the case of R.A.F. Section 31 wings, however (a thick, nearly symmetrical section), the wind channel readings give a lift coefficient of about 0.53, whereas readings taken in flight on a full scale machine fitted with wings of this section prove that a lift coefficient of 0.64 is obtained—a difference of 0.11, and a very appreciable error. There is reason to suppose that with a thick symmetrical section the error will be even greater.

Experiments are therefore being carried out in an attempt to reach some definite conclusion with regard to these errors due to scale effect.

Several full scale machines have been fitted with wings of different sections, as well as with Handley-Page slots, Fairey flaps, and other devices. Readings are taken on these machines in the air, and the lift and drift coefficient deduced and compared with those measured on their models in the wind channel. It is obvious that unless the conditions of flight of the full scale machines correspond to those of the models, the observations taken will be useless for comparison. For this reason these flights are carried out in the finest of weather—in summer in the very early morning, when the air is undisturbed by the effect of the heat of the sun, or by wind and clouds. A band-brake is fitted to the hub of the airscrew, by means of which it can be locked in the same position as the airscrew on the model. This is done to eliminate the use of elaborate mechanism in the wind channel to rotate the airscrew on the model at the appropriate speeds. In order to obtain accurate airspeed readings, the pressure head of the standard pitot tube is fitted with a tail and wind-vane, and is free to swivel up and down, the action of the air on the wind-vane thereby holding the pressure head dead into the relative wind. The static head of the airspeed indicator is suspended 40 or 60 feet below the machine at the end of a rubber tube, its weight being taken by a thin wire cable threaded through the inside of the tubing. The static head itself projects directly forward into the relative wind, held in this position by means of a tail fitted with vanes in the same manner as the swivelling pressure head. The object of suspending the static head is to ensure accurate readings, free from any local disturbances caused by the machine itself. The pressure head

is not affected by local disturbances in the same way, and need not, therefore, be suspended clear of the machine.

The experiments are carried out in the following manner:—The machine is climbed up to a height of eight or ten thousand feet. On the way up the observer lowers the suspended static head over the side, and takes temperature readings every thousand feet from a thermometer fixed on an interplane strut. When this height has been reached, the pilot switches off the engine and locks the airscrew. The machine is then held in a glide as steadily as possible at a certain pre-arranged speed. At slow speeds—just above the stall—it is possible with practice to keep within half a mile an hour each way of the given speed; but at high speeds, when the glide becomes so steep that it may almost be called a dive, the effectiveness of the elevator control becomes so great that it is a very difficult matter to fly accurately—the slightest movement of the control column causing a considerable alteration in the altitude and speed of the machine. Most pilots agree that the difficulty of maintaining a steady speed is considerably lessened if the machine is slightly nose heavy, so that it requires "holding up" to the required speed in the glide by hooking the fingers round the front of the control column. This seems to give a greater sense of "feel" than if the machine were tail heavy or correctly trimmed. This nose-heaviness cannot, of course, always be arranged.

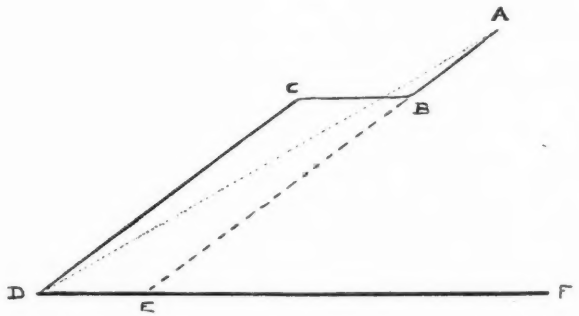
Gliding accurately beyond the stall is no easy matter, as the controls lose so much of their effect that the machine is very liable to "wallow," and unless the pilot is very attentive, it will "drop out of his hand" and dive for some distance before he can regain control.

As soon as the speed is steady, the observer makes a note of the altitude as registered by an accurate aneroid (or altimeter) graduated to every 20 feet and starts a stop-watch. An accurate variable inclinometer is fitted and readings of angles of inclination, altitude, airspeed and time are taken the whole way down the glide. When the machine has descended to within two or three thousand feet of the ground the band-brake is released, the machine dives steeply until the airscrew rotates, and the engine restarted. The machine is climbed once more, and another glide carried out at a different speed.

The total weight of the machine before flight, including pilot, observer, petrol, oil and water, etc., is known. The total time the engine has been run is noted by the observer during flight, and the consumption of the engine being known, the weight of petrol and oil can be calculated. The weight of water lost during the flight is calculated from the amount of water taken to refill the radiator. The temperature at different heights has been observed, so the density of the air at different altitudes can be computed. From observations taken on the glide the following readings are recorded:—Speed, attitude of machine to horizon, difference in altitude, and times. From these data the coefficients of lift and drag are obtained without much difficulty.

It will be realised that this method of obtaining the coefficients of lift and drag lays itself open to serious errors due to ascending and descending currents of air which cannot be measured and allowed for in the calculations. If a machine on the glide encounters a rising current of air, this rising current will tend to prolong its glide. It must be remembered, however, that the angle of the machine to the horizon remains the same. The inclinometer readings will remain constant, but readings of time, speed and altitude will no longer give the correct gliding angle in undisturbed air.

Suppose the aeroplane starts its glide at A. At B it encounters a rising current of air, which deflects its flight path from what would have been AE to ABC. At C it has passed through the rising current into still air again, and glides down to D, parallel to AE, but continued further in horizontal distance owing to the influence of the rising current.



The *true* gliding angle, it will be seen, is the angle AEF. But according to the flight path of the machine the *apparent* gliding angle is ADF—a much more acute angle.

The same applies, of course, to a descending air current, only in this case the gliding angle will appear to be greater.

As these ascending and descending air currents occur frequently on apparently still days, and as they are so steady in their motion that they cannot be felt in the machine, some method has to be adopted whereby the consequent error can be eliminated.

To overcome this difficulty an ingenious piece of mechanism known as the suspended flight path recorder has been designed, and has proved itself in every way satisfactory on its trials. The instrument consists of a torpedo-shaped casing, from the nose of which projects a pressure head within a static head—forming a complete pitot. Vanes are fitted to the tail of the bomb-like case. Inside this casing is an electric motor, driving a drum on to which is wound a length of cinematograph film at slow speed. On to this film is projected a pin point of light, reflected from a mirror and generating from a small electric lamp. The mirror is fixed to a pendulum (suitably fitted with damping springs). Another pin point of light is projected on to the same film and reflects from a glass fibre attached to the diaphragm of an airspeed indicator, which is operated by the static and pressure heads protruding from the nose of the instrument. The flight path recorder is lowered some 100 feet below the machine. The vanes on its tail ensure it taking up the true gliding angle of the machine, which—with the airspeed—is recorded on the film.

Whatever altitude variation may occur in the path of the gliding aeroplane, the true gliding angle and airspeed are thereby recorded in this instrument.

In order to obtain certain data it is necessary to record the angles of the different control surfaces in flight. This is done by means of curious-looking instruments officially known as control movement recorders, but more commonly alluded to as "Rats." (The strange nickname is no doubt due to the unusual appearance of the instrument—a small black object with a long tail.)

The streamlined casing is fixed to a span or rib immediately in front of the control surface. The tail—a brass rod—is attached to that surface, and when the surface moves the tail (by a simple system of levers) tilts a reflecting mirror inside the casing. A small windmill drives a drum on to which is wound a length of film, and its speed is regulated by centrifugal governors, and it is started or stopped at the will of the pilot, or observer by means of an electric clutch worked from the cockpit by a tumbler switch, which also switches on or off the small electric bulb. The various angles through which the control surface is moved are thereby recorded on the film in very much the same way as the airspeed and gliding angle are recorded in the flight path recorder.

In order to measure the forces on these control surfaces, a device known as the force-recording control column is used. This consists of a double control column, one within the other. The inner one is mounted on springs within, and projects above, the outer casing. It is balanced on gimbals, and fitted with a screw-locking device, so that the two columns can be locked together. At the base of the double column is a box containing the usual film, bulb and reflector. When it is desired to measure the forces on the elevators or ailerons, the locking device is unscrewed, the film started, and the top of the inner column held. The reflector, being attached to the base of the inner column, is tilted as the forces act on the column against its springs, and a record of their magnitude is taken on the film.

A force-recording rudder-bar is at present under construction. This will measure the forces acting on the rudder.

One of the greatest causes of serious and fatal accidents in flying is the spin as a result of a stall. This applies particularly to training.

Until comparatively recently very little was known about the theory of the spin, and owing to the fact that it is a very complex evolution involving moments in the yawing, looping and rolling planes, the successful imitation of it in the wind channel has up to the present date defeated all attempts to investigate it as it really is. The various movements have been split up and dealt with separately, but unfortunately the sum totals do not agree with the results obtained on a full scale machine.

The wind channel experts are determined to master the difficulties by obtaining readings from a spinning model, and it is hoped that before long this will be accomplished. It will surely be a great stride in the progress of aeronautical research.

In the meantime experiments are being carried out on a full scale machine —a veritable Christmas tree of "gadgets." On this machine it is possible to take readings while spinning, of the angles of the different control surfaces, degree of side-slip on each wing, accelerations in three planes, rate of descent, and rate of rotation. It is hardly a comfortable experiment to carry out—it is even worse for the observer than it is for the pilot. A spin at the best of times is rather a bewildering affair, but to have to take numerous readings from instruments scattered all round the cockpit in a prolonged spin is indeed working under difficulties.

An airscrew brake is fitted, as in the machines used for gliding experiments, and for measuring the side-slip a wind-vane is fitted on an outrigger ahead of each wing. By means of small resistance coils attached to these wind-vanes and connected to millivoltmeters in the cockpit it is possible to read the side-slip on each wing accurately. The observer also has in his cockpit an adjustable inclinometer (for registering the direction of acceleration in the looping plane), an accurate altimeter, a stopwatch, and a lateral level (for checking the lateral accelerometer).

In the pilot's cockpit, placed as nearly as possible to the centre of gravity, are two accelerometers, one normal and one lateral. The pilot also carries with him a stopwatch.

"Rats" are fitted to each lower wing-tip, their "tails" attached to the ailerons; to the tail plane, for attachment to the elevators; and to the vertical fin, for the rudder.

The accelerometers used record the accelerations, normal and lateral, throughout the spin. A ray of light is reflected from a fine glass fibre on to a moving film. The fibre is semi-circular, and mounted on a fixed base. The normal accelerometer is placed so that the plane of the fibre lies horizontally and athwartships. The lateral one is placed so that the plane of its fibre is in a vertical and fore-and-aft position. As only very small lateral accelerations

occur in a spin, its fibre is especially made, and has a diameter of about half a thousandth of an inch. It registers differences in acceleration between about +0.15 g. and −0.15 g. for the whole width of the film. The normal accelerometer registers variations between −2 g. and +3 g. for the whole width of the film.

In this experiment the sequence of operations is as follows:—

The machine is flown up to five or six thousand feet. The pilot then operates the levers starting the films and switching on the lights in the accelerometers. The engine is switched off and the airscrew locked. The machine is stalled by easing back the control column, and dropped into a spin by kicking full rudder on in the required drection. Some prominent object is sighted on the ground—such as a dusty road, which shows up distinctly from the air. When the machine is spinning properly (after one or two turns) the pilot takes the time to complete five more turns, with his stopwatch. This is done by watching the radiator filler cap, or some such point on the nose of the machine, flash past the object previously sighted on the ground. (It has been found that the rate of rotation can be surprisingly accurately obtained in this way.)

The observer in the meantime has been working feverishly recording the time for a given height-drop, the position of the lateral bubble, and as many readings as possible of the side-slip indicators and adjustable inclinometer—to say nothing of doing his best to keep down his last meal.

Our rodent friends, the control movement recorders, have been busy recording the angles of ailerons, elevators and rudder, and the accelerometers have also done their bit in recording accelerations.

About ten complete revolutions have been completed in some 25 seconds, and approximately three thousand feet have been lost in height by the time the machine is brought out of the spin and has recovered from the resulting dive. During the spin the force of gravity has been increased to 2½ to 3 times normal. In other words, every part of the anatomy of the entire ship's company weighs nearly three times as much as it would normally. Blood is thrown from head to feet, and the brains are subjected to—heaven knows what forces and pressures. It is a considerable effort to think clearly, and as the medical officer would put it, "a distinct feeling of nausea is experienced."

Experimental flying from the pilot's point of view is in these circumstances a dizzy affair, to put it mildly.

I will not attempt to explain to you how all the readings taken on this experiment are dealt with by the "X-chasers" to produce useful data; the spin is far too complicated an evolution to go into thoroughly without becoming technical. It may be of interest to some of you to hear that the angle of incidence in a spin is sometimes in the region of 60 degrees. It will be fully realised then that unless an adequately placed and reasonably sized rudder is fitted, its effect is liable to be blanked out completely by the tail-plane and fuselage.

(2) *Control at Low Speeds.*—In order to overcome the difficulties of maintaining control beyond the stall, various experiments have been carried out on full-size machines by means of fitting experimental control surfaces.

A standard Avro 504 training machine was fitted with a large fin and rudder. In this case it was found that the machine could be held very steadily in a stalled glide, and that it was possible to influence the character of a spin considerably by use of the rudder. By putting it on full in the opposite direction the machine could be brought out of a spin in spite of the fact that elevators and ailerons were still in spinning position.

Another standard Avro is fitted with a slot and aileron control, the Handley-Page slot in the leading edge opening and shutting in conjunction with the aileron moving down and up. This machine can be flown stalled with engine

on and off, and although the nose may drop slightly when in the stalls, perfect lateral control is maintained throughout. It seems quite within the realms of possibility that provided a suitably strong undercarriage is fitted to withstand the shock, a stalled landing might be carried out successfully from any height without serious damage to machine or personnel; but this has not been tried up to date. It has been found that this machine can be brought out of a spin by use of the slot and aileron control only, with elevators and rudder still in spinning positions. This is a great advantage in training machines, where the pupil, finding himself in a spin, is very liable to lose his head temporarily and clutch the control column, holding it back in a vain attempt to bring the nose up. This loss of self-control may cause him the loss of his life, as in most machines pulling the control column back keeps the machine stalled, and holds it into the spin.

A standard Bristol Fighter has recently been fitted with this device, and it is hoped that this aeroplane will prove itself as foolproof as the modified Avro.

Later, it is intended to combine the slot and aileron control with the large fin and rudder. The machine fitted with these should prove interesting.

Another experiment was carried out with the directional control of a De Havilland " 10," a twin-engined machine with monoplane tail and single rudder. It was found that the rudder control was very heavy and sluggish, and inclined to lose its effect at low speeds.

The rudder was therefore modified in the following manner:—Two spars were fitted to a rudder not unlike the standard one. These spars projected nearly eight feet behind the main rudder hinge, and a small " Jury " rudder was fitted between these two outriggers, and connected by control wires to a small rudder-bar superimposed upon the main one.

The control wires to the " Jury " rudder were crossed so that when its rudder-bar was kicked on to the right, the " Jury " rudder itself was put on in the opposite direction (*i.e.*, to the left). The effect of the wind on the " Jury " rudder was to carry it over to the right, and its leverage was so great that it carried the main rudder over to the right with it, thus turning the machine to the right.

So light and effective was the rudder control on this machine after this experiment that it was found necessary to fit shock-absorbing rubber cords to make the control heavier, in order to make the machine safe to fly!

Engines

With regard to experimental work on aero engines, there is not a great deal that can be said in this paper. The subject is such a large one that it really deserves a paper to itself. Unlike aeroplanes in their experimental stage, engines can be tested and tried out on the ground to a large extent, although there are certain experiments which have to be carried out in the air, such as experiments at high altitudes, which cannot very well be carried out anywhere but in machines flown at those altitudes.

These experiments are nearly all to do with supercharged engines in their experimental stages, and the supercharged engine and high altitude flying are subjects too large to deal with in this paper. A few general remarks, however, may be made in regard to this work.

High altitude work involves the use of oxygen and heated clothing. Both are liable to fail at times, and the pilot is on those occasions left in a very uncomfortable position. Temperatures of the atmosphere at 30,000 feet (close on six miles high) often run as low as −60°C., and unless electrically-heated clothing is working well, the pilot is liable to suffer great discomfort from the cold, and is not infrequently threatened with frostbite.

There is a very well-known test pilot who left the Royal Air Force less than a year ago who is an expert at high altitude flying (among other branches of the art of which he is undoubtedly a master). An experience of his will serve to illustrate the little difficulties that arise when the oxygen fails far better than I can hope to explain to you in pages of writing, for his is a true story, and fact is stronger than fiction.

He was flying along at a great altitude—not far under 30,000 feet—taking observations from various instruments. The next thing he was conscious of was that he was some 10,000 feet lower! His oxygen had failed, and without the slightest warning he lost consciousness, and did not come to his senses until he had descended thousands of feet. It can quite well be imagined that such an occurrence as this might end disastrously.

High altitude flying is a tiring occupation—as indeed is a lot of work on experimental engines. Although the lungs are incapable of taking in too much oxygen, two or three "doses" in one day brings on a strange reaction towards evening, and has a very enervating effect.

Experiments in connection with different types of radiators, both for water and for oil, have to be carried out in the air, and high altitude flying is also necessary with trials of different fuels and oils. Engine research work naturally calls for a high standard in piloting because experimental engines and their accessories have a habit of being rather unreliable, and the pilot frequently finds himself placed in an embarrassing position with a "dead" engine. Supercharged engines especially, in the experimental stage, have a habit of failing to supercharge at high altitudes. In these cases the pilot is often left with several thousand feet to glide before passing through a cloudbank and before he is able to see the ground.

It is very pleasing to note that designers are paying more attention to those little details which make the cockpit comfortable for the pilot. It is most essential that everything is placed within easy reach, or in the case of instruments where they can be easily seen. Given two machines of equal performance, the one with the pilot's seat, the controls, wind-screen, and instruments nicely arranged, and the other with no attention paid to the comfort of the pilot, it is surprising how much more pleasant it is for him to carry out his work—whatever it may be—in the former machine, and how badly it will be done in the latter.

Although the pilot's seat be ever so nicely padded and made like an armchair, and however fine the weather and favourable the conditions, there is always a considerable nerve-strain in flying, and the pilot is not allowed to relax himself for an instant the whole time he is in the air, and three or four hours' flying in one day is as tiring as a considerably longer stretch at the wheel of a car.

ASSOCIATE FELLOWSHIP EXAMINATION

STRENGTH OF MATERIALS AND THEORY OF STRUCTURES

Time allowed: Three hours. *Eight questions to be attempted.*

1. Define the following terms: Elasticity, Modulus of rigidity, Poisson's ratio, Stress at a point.

2. Describe in detail how you would carry out a tensile test on a specimen of round bar steel to determine the limit of proportionality, yield point, Young's modulus, breaking stress, etc. Discuss the best form of test piece and grips and the method you would use to measure the strains.

3. How would you determine Young's modulus for a specimen of timber? Explain the reason for any special precautions you would take.

4. N points in space are to be connected together by rods to form a simply stiff frame. How many bars are required?

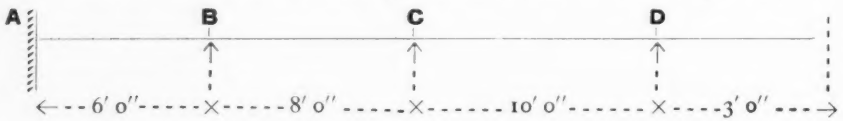
   If P of the N points are attached to a rigid body how many members are needed to brace the remaining points to these P?

5. A beam of 20 feet span is built in at both ends and carries:—
   (*a*) A uniformly distributed load of $\frac{1}{2}$ ton per foot.
   (*b*) A concentrated load of 2 tons at 5 feet from the left hand support.
   (*c*) A concentrated load of 1 ton at the centre.

   Calculate the fixing moments at each support. Prove that the method you use is correct.

6. Outline the properties required in steel strip used for the construction of aeroplane members and describe the tests used to ensure suitability.

7. The continuous beam shown is built in at A and simply supported at B, C and D. It carries a uniformly distributed load of 100 lbs. per foot.



   Sketch the bending moment and shearing force diagrams.

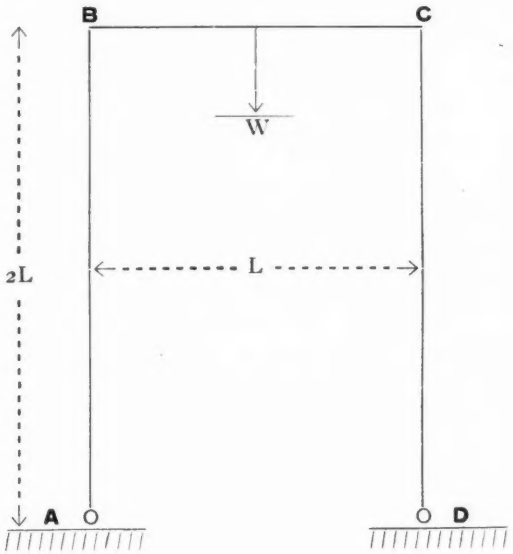
   If the beam were subjected to axial compressions in addition to the lateral loads indicate generally the effect upon the results.

8. A rectangular beam of silver spruce is twice as deep as it is wide. It is freely supported at the ends of its span of 10 feet and carries a uniform load of 100 lbs. per foot and an axial load of 5,000 lbs. Determine the size of the beam if the stress is not to exceed 5,000 lbs. per sq. inch.

   $E = 1{,}600{,}000$ lbs. per sq. inch.

9. Describe a method for calculating the stresses in a fuselage of the usual braced type due to the action of air loads on a fin and rudder set above the axis of the fuselage.

10. The frame shown is pinjointed at A and D and the joints at B and C are rigid. A load W is carried at the midpoint of BC. Draw the bending moment diagram for the frame (all members of which are of the same section).



MATHEMATICS

Time: Three hours. ***No more than Six questions need be attempted.***

1. A model of an airship is so constructed that the nose is a hemisphere of radius a fitted to one flat end of a circular cylinder of radius a and length l while the tail is a right cone of circular base of radius a fitted to the other end of the cylinder and of angle 2α at the vertex. Determine the Moment of Inertia of this solid model about the longitudinal axis.

2. The velocity v of a particle, after traversing a distance s is given by

$$v^2 = V^2 (1 - s^2)$$

where V is a constant. If time is measured from the instant when s is zero determine the acceleration of the particle and the distance traversed after any time t.

3. A triangular framework is composed of three equal heavy bars each of length a and weight W, freely jointed together at their ends. A string of length $2a$ has its ends attached to the pins at two joints and is hung evenly over a smooth pulley. Determine the reactions at the pins.

4. Explain what you understand by the statement that a physical equation must be dimensionally homogeneous.

A series of struts geometrically identical, differ only in scale. Show that if failure occurs owing to instability under longitudinal end-thrust the latter will be proportional to EI/P, where I is the length of any member of the series, I is the inertia of its end section and E is the elastic modulus of the material.

5. If a real root of an algebraic equation be known, approximately describe any method whereby the accuracy of the root may be progressively increased.

Determine the real root of the equation

$$x^3 + 5x - 4 = 0$$

Correct to three decimal places.

6. Prove that all the roots of the equation

$$x^4 + 0.1x + 16 = 0$$

are complex and determine them to two significant figures.

7. Determine the nth differential coefficient of $e^{-x} \sin x$. Hence or otherwise expand $e^{-x} \sin x$ in ascending powers of x.

8. Evaluate

$$(1) \int dx/(1+x^2)^{3/2}\,; \quad (2) \int_0^{\pi} \sin\ \theta d\theta$$

9. Draw a rough sketch of the curve

$$y = (\sin x)/x$$

and deduce that of

$$y^2 = (\sin x)/x$$

AERODYNAMICS

Time allowed: Three hours. ***Answer Six questions only.***

1. Neglecting compressibility, obtain a general formula for the drag of geometrically similar bodies moving through a fluid, in terms of the density and viscosity of the fluid and the size and velocity of the bodies. Give simplified forms of your general formula that would hold good approximately for smooth round wires (*a*) of about 0.01 in. diam. moving at about 5 ft. per sec. through air, (*b*) of about 0.2 in. diam. moving at about 100 ft. per sec. through air, the pressure and temperature having normal ground level values. What essential difference in the character of the fluid motion would you expect to find in the two cases?

2. Write a brief account of the mechanism, according to the Prandtl aerofoil theory, by which an aerofoil in a moving fluid derives lift. Confine your attention to the two-dimensional case and include in your answer a definition of "circulation."

3. Give the theory of the pitot-static pressure head. Describe how it is used to measure the speed of an aeroplane, dealing in detail with calibration for disturbance due to the wings. Indicate the results you would expect from such calibration, stating the position on the aeroplane selected for the instrument. How is the true velocity at considerable altitude related to the indicated air speed? Why is it more suitable to employ this instrument than one indicating true velocity?

4. A light aeroplane has a maximum speed of 62 m.p.h. when its overall weight is 600 lb. The power unit develops 15 t.h.p. The wing system, which totals 210 sq. ft. in area, yields the following test figures:—

k_L	...	0.1	0.2	0.3	0.4	0.45	0.56 (max.)
L/D	...	13.0	18.0	15.7	13.0	11.7	9.0

Find approximately the maximum additional load the aeroplane could carry in horizontal flight.

5. An aeroplane weighing 2 tons has an overall lift-drag of 8.0 at a low altitude cruising speed of 80 m.p.h. While flying at this speed, the throttle of the engine, which develops 450 b.h.p., is fully opened. At the same time the angle of incidence is maintained at its original value by means of the elevators. Assuming the airscrew efficiency to be 72 per cent., find without unnecessary approximation the resulting rate of climb.

6. Both upper and lower wings of a biplane are 40 ft. span and have a dihedral angle of 5°. The total wing area is 500 sq. ft. and the lift-angle curve is straight between $-2°$ ($k_L=0$) and $+10°$ ($k_L=0.420$). While flying at 100 m.p.h. (2° incidence) the machine encounters a side gust of 15 m.p.h. Estimate by approximate calculation the initial value of the rolling moment introduced.

7. Show that the lift of an airship is independent of slow changes in altitude so long as the hydrogen containers are permitted free expansion. Discuss the case of rapid changes in altitude and explain how stability in respect to these in an average state of the atmosphere arises. (Neglect variation in density due to size of envelope.)

8. Obtain the following formula for the inflow factor (a) at a radius γ of an airscrew :—

$$a/(1+a)=(\lambda/2\pi)\,(C'/\gamma)\,(f(a))\,(\cos\{\phi+\gamma\})/\sin^2\phi$$

where λ is the ratio of the inflow factor to the outflow factor; C' is the sum of the chords, $f(a)$ the coefficient of resultant force, ϕ the direction of the wind relative to the element, and γ the angle between the resultant force and the lift of the element—all at radius γ.

9. An airscrew turning at 1,500 r.p.m. drives an aeroplane at 100 m.p.h. At a radius of 3 ft. 6 in. the lift/drag of the element is 18.0 and the local efficiency 75 per cent. Find the thrust per ft. run at this radius on each of the two blades. (Take 0.35 for the ratio of the inflow factor to the outflow factor.)

10. Explain in detail, illustrating your answer by means of curves or approximate calculations, how the location of the centre of gravity of an aeroplane with respect to the wings affects stability and sensitivity to longitudinal control.

ROYAL AERONAUTICAL SOCIETY

Reception in Honour of Sir Alan Cobham, K.B.E., A.F.C.

A reception was held on Monday, October 25th, 1926, at the King Edward VII. Rooms, Hotel Victoria, in honour of Sir Alan Cobham. The reception was preceded at 7.15 by a Council dinner at which Sir Alan and Lady Cobham were the chief guests.

At the reception nearly 400 guests and members of the Society were received by the Chairman, Colonel the Master of Sempill. Among those present were some of the Dominion Prime Ministers and delegates to the Imperial Conference, the London representatives of British Dominions, the Duke of Sutherland, most of the leading members of the aeronautical industry and the Council.

The reception was immediately followed by a lecture by Sir Alan Cobham on his flight from England to Australia and back, illustrated very fully with lantern slides. A full report of this lecture will be given in a later issue of the Journal.

Before introducing the lecturer, Air Vice-Marshal Sir Sefton Brancker, President of the Society, announced that at a Council meeting Sir Alan Cobham had been elected Honorary Fellow of the Society, the highest grade the Society has to offer, and Sergeant Ward and Mr. Capel to Life Associateship.

The following letter was read by the President from H.R.H. the Duke of York:—

17, Bruton Street,
Mayfair, W.1,
23rd October, 1926.

Dear Sir Sefton Brancker,

I very much regret that I will be unable to be present at the Royal Aeronautical Society's reception on October 25th.

I am proud to feel that the Prince of Wales and I share between us the honour of being Patrons of the oldest aeronautical body in the world, and it would have given me the greatest satisfaction to be able to attend the reception.

The work of the Society is growing constantly in magnitude and importance. Its position can be maintained and improved only if all those connected either directly or indirectly with Aeronautics give their fullest support. The munificent gift by the Guggenheim Trust Fund of the U.S.A. will in a measure enable the scope of the Society's activities to be widened, and I hope that those public-spirited persons or bodies who have in the past done so much to help forward our other great societies will give assistance so that a substantial Endowment Fund may be established.

Yours sincerely,
(Signed) Albert.

The lecture was followed by a dance and one of the most successful evenings ever held by the Society ended at midnight.

The Council wish to place on record that they feel much of the success of the evening was due to the indefatigable energy and organising ability of their Chairman, Colonel the Master of Sempill and his Secretary, Mr. Brutnell.

REVIEWS

Aircraft Instruments

By H. N. Eaton, K. H. Beij, W. G. Bronnbacher, W. W. Frymeyer, H. B. Henrickson, C. L. Seward and D. H. Strother. 269 pages, 58 line drawings and 10 plates. Published by the Ronald Press Co., New York, price 5 dollars.

Although fairly extensive information on aircraft instruments has been published in the Reports and Memoranda of the Aeronautical Research Committee, in the American National Advisory Committee for Aeronautics Reports and in the technical press, the authors of *Aircraft Instruments* have achieved the distinction of being the first to prepare, in the English language, a comprehensive treatise on aircraft instruments in book form.

The especial merits of the book and certain defects have a common origin. The system of "mass authorship," involving the combination of the work of seven writers, all employed in the Aeronautic Instruments Section of the U.S. Bureau of Standards, has resulted in the compilation of a wealth of reference data, but the value of the work as a text-book is, almost inevitably, inferior. It is hardly possible to refer to, or to describe, as does this book, some 160 different instruments and at the same time to provide a satisfactory account of the fundamental principles of their operation without rendering the work prohibitively large.

Chapters I. and II. describe a number of instruments which have been developed for the measurement of altitude, rate of climb, air speed and ground speed. Chapters III. and VIII. deal with navigational instruments, including turn indicators, pitching indicators and yawmeters. Power plant instruments are described in Chapter IV. and a number of types of manometer in Chapter V. Chapter VI. describes some special instruments for airships, including fabric tension meters and gas leak detectors. Measurements of air and gas temperatures are dealt with in Chapter VII. and oxygen supply apparatus in Chapter IX. Chapter X. describes some special test instruments. Several tables are given showing the weights of various types of instruments developed in America and in other countries.

If the book cannot be unreservedly recommended as a text-book for aeronautical students, it can be strongly recommended as a reference book for those engaged on investigations on aircraft instruments. Moreover, the book presents a very clear picture of the state of development of this subject in the United States.

The Metallography of Steel and Cast Iron

By W. E. Woodward. Published by Crosby Lockwood, price 15/-.

This book will no doubt be useful to first or second year students, particularly those who cannot for some reason find the time to attend the lectures from which it has been "put together." As, however, it is evidently intended for students, it would appear to be advisable to avoid such generalities as occur on page 39, for instance, where it is stated that "steel containing about 0.12 per cent. P . . . is used for nuts and bolts." If the author finds it absolutely necessary to refer to nuts and bolts, this should read "sometimes but rarely used, etc.," to avoid the possibility of another of the many "howlers" which are traceable

to the impressions left by the wording employed by otherwise well-intentioned authors.

As a book such as this is often read by advanced students and engineers, though it is not written primarily for them, it seems a pity that the references to the original work of the authors quoted is not given, particularly as the extra trouble required is small compared with the increased usefulness. For example, when the different theories of hardening are discussed, no reference is given to the papers where these theories are presented by the various investigators. The bibliography is also poor, and advises the student who wishes to study the subject more fully to look for the references he requires in one of the other books on the subject.

The main portion of the book presents the photomicrographs of some typical steels and irons which are considered in conjunction with the necessary equilibrium diagram, and in so much as the author has made his subject appear simple, will serve a useful purpose.